# The Hierarchy Problem

What’s wrong with the Higgs boson?

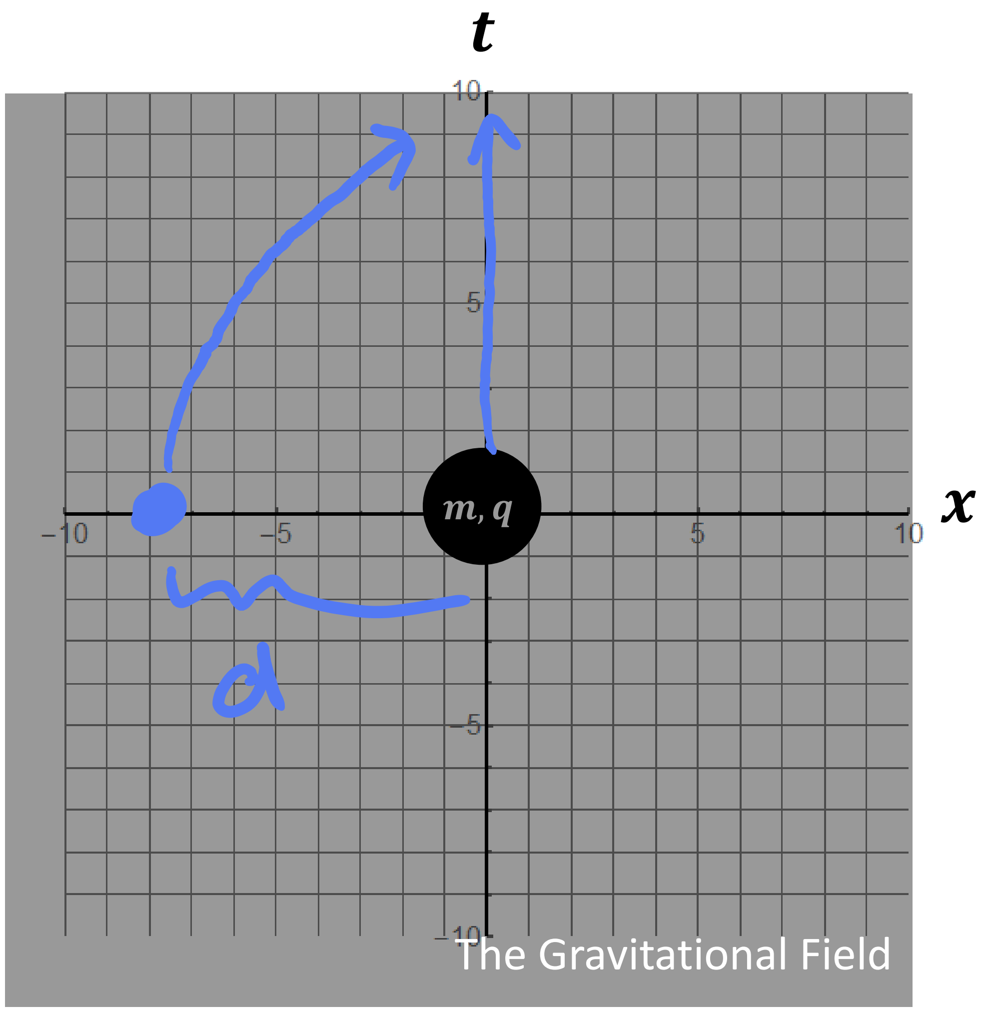
# Ingredients: Spacetime

- The gravitational force field

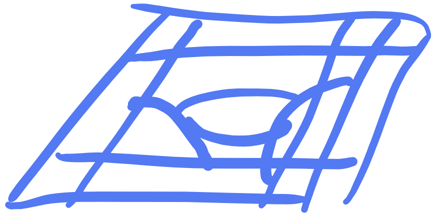
$$V_G(r) = -\frac{G_N\,m}{r}$$

$$V_G(r) = -\frac{G_N m}{d}$$

$$\frac{d}{dr}V_G(r) = F(r)$$



# Ingredients: Spacetime

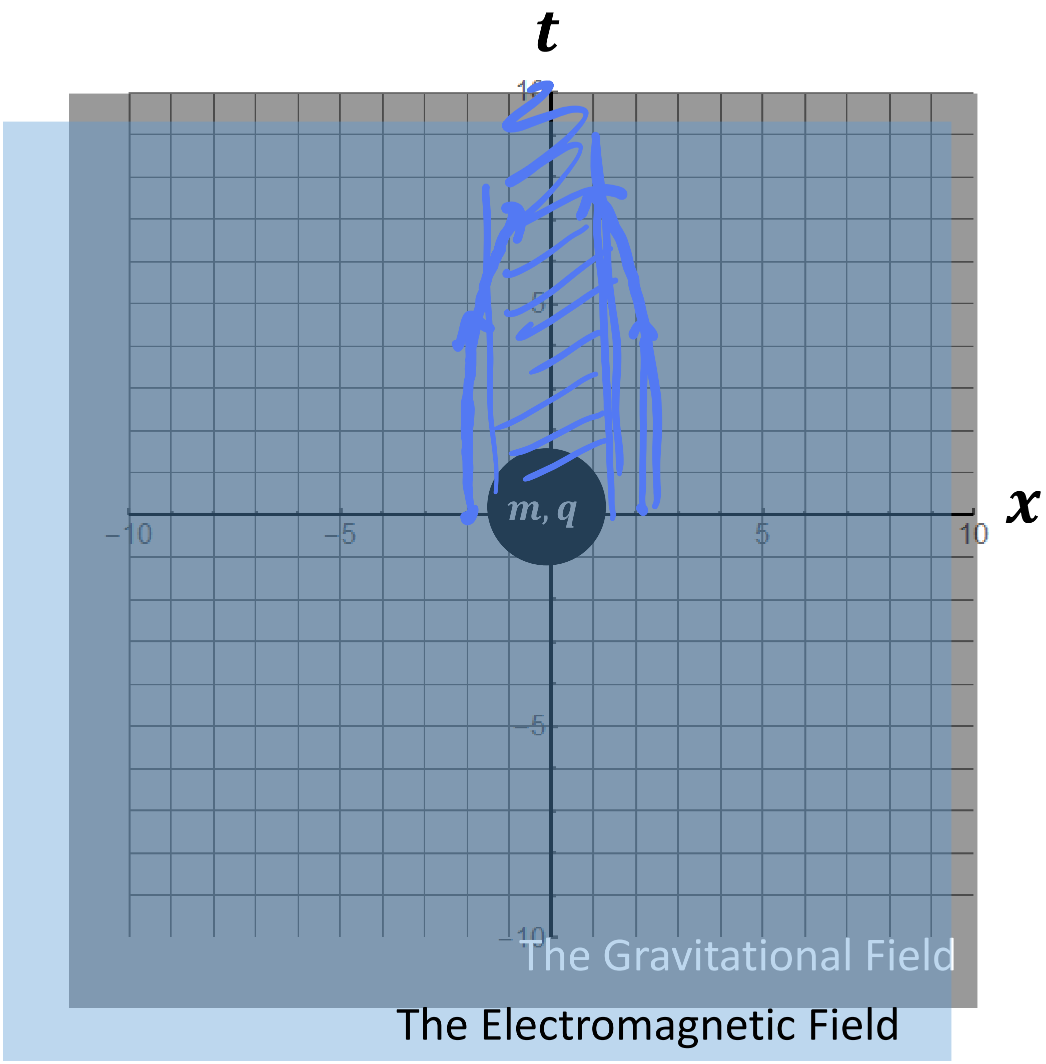


- The gravitational force field

$$V_G(r) = -\frac{G_N m}{r}$$

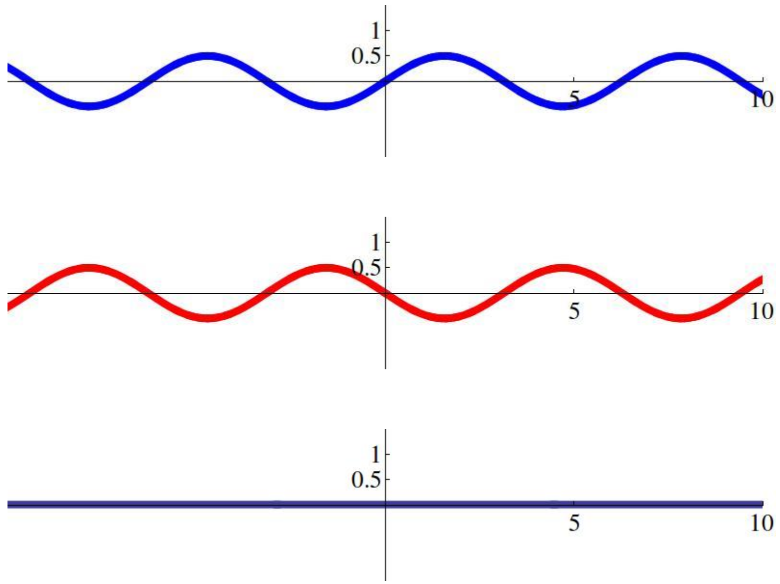
- The electromagnetic force field

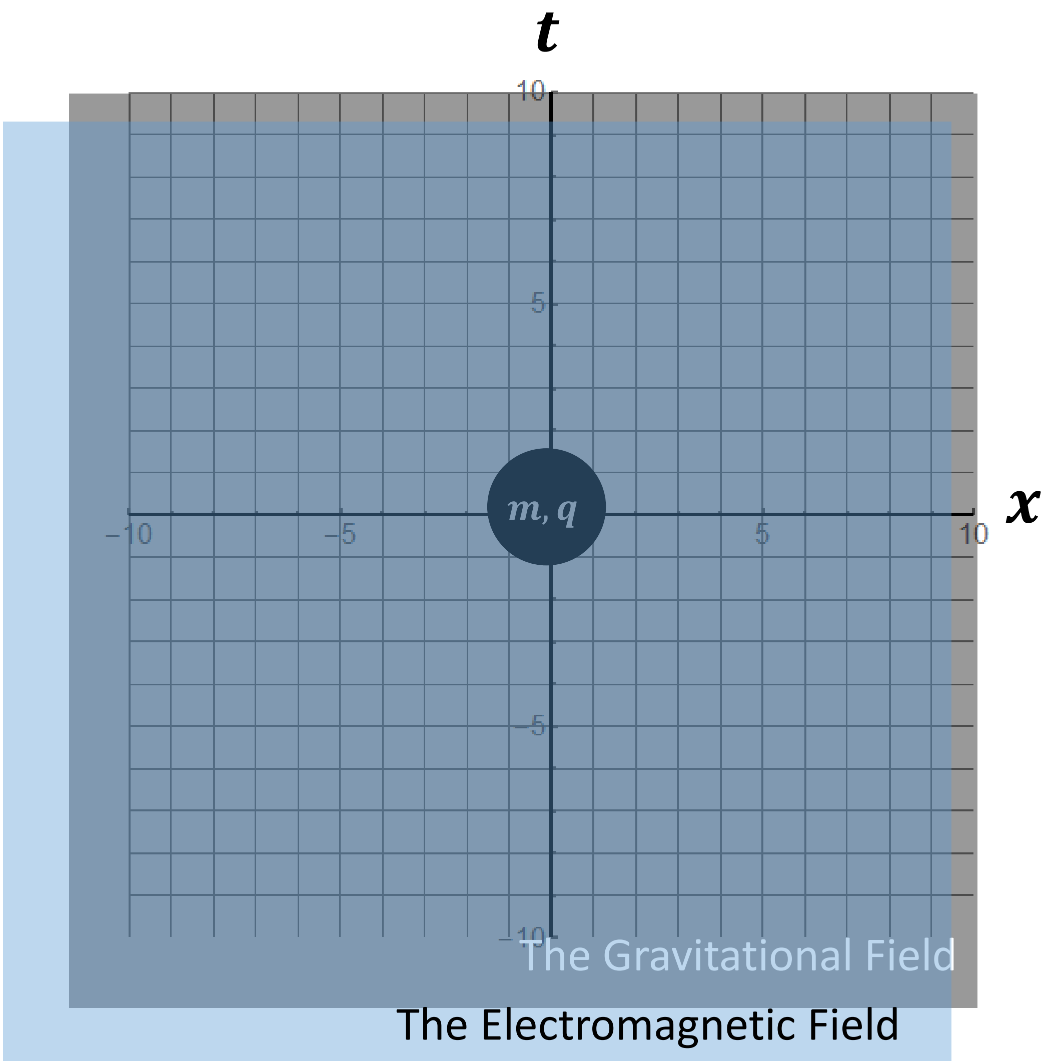
$$V_{EM}(r) = -\frac{k_e q}{r}$$



# Ingredients: Spacetime

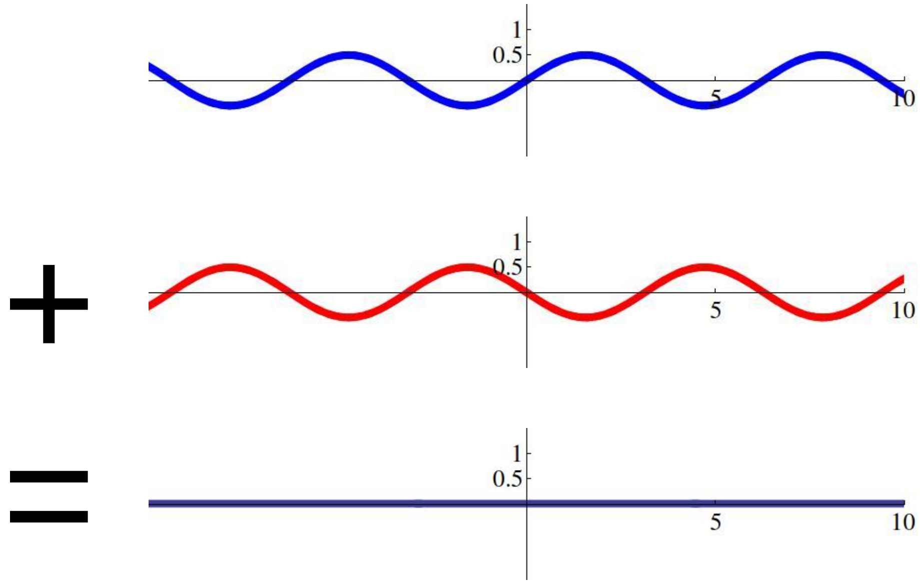
- Fields can move energy through space in the form of propagating waves





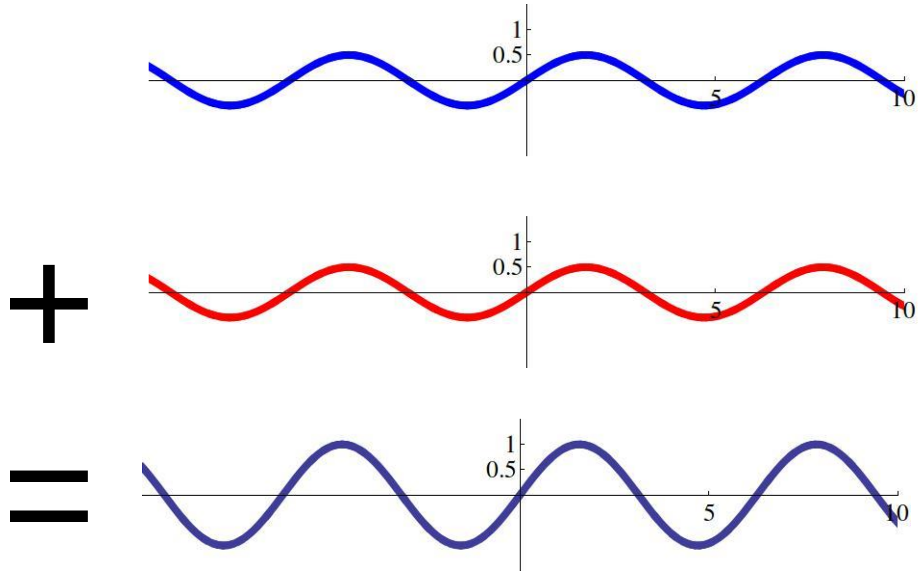
# Ingredients: Spacetime

- Fields can move energy through space in the form of propagating waves
- Waves can be added to cancel each other out



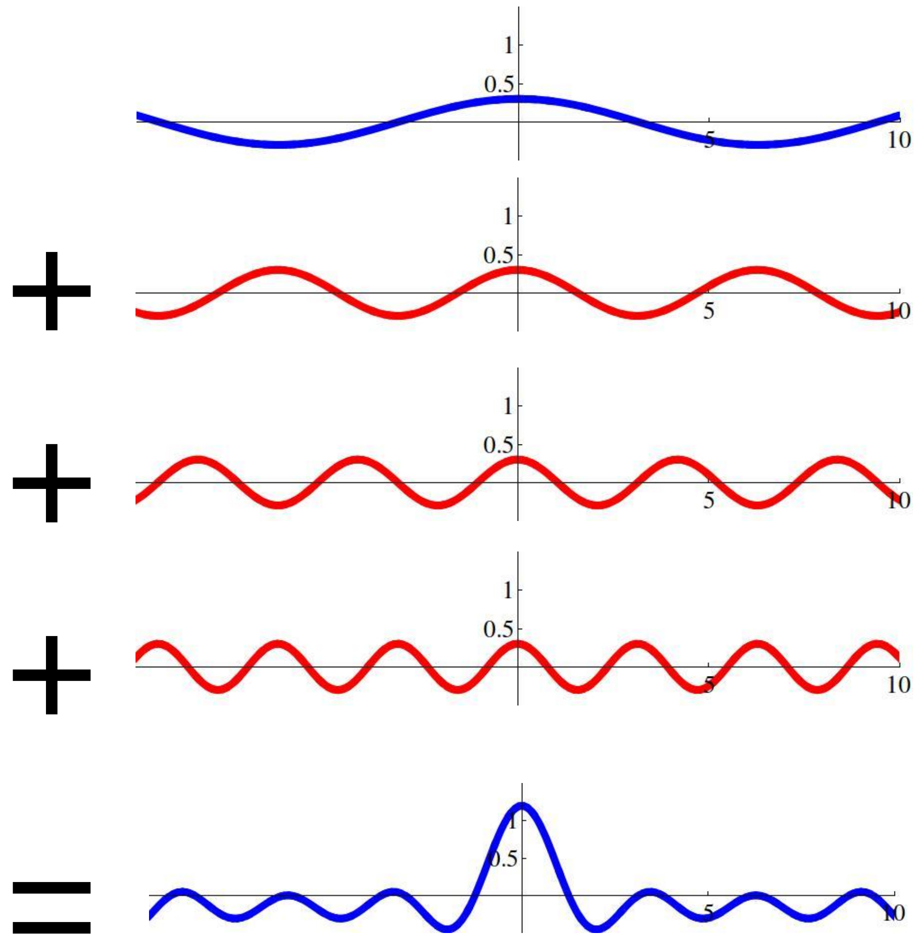
# Ingredients: Spacetime

- Fields can move energy through space in the form of propagating waves
- Waves can be added to cancel each other out… or magnify each other



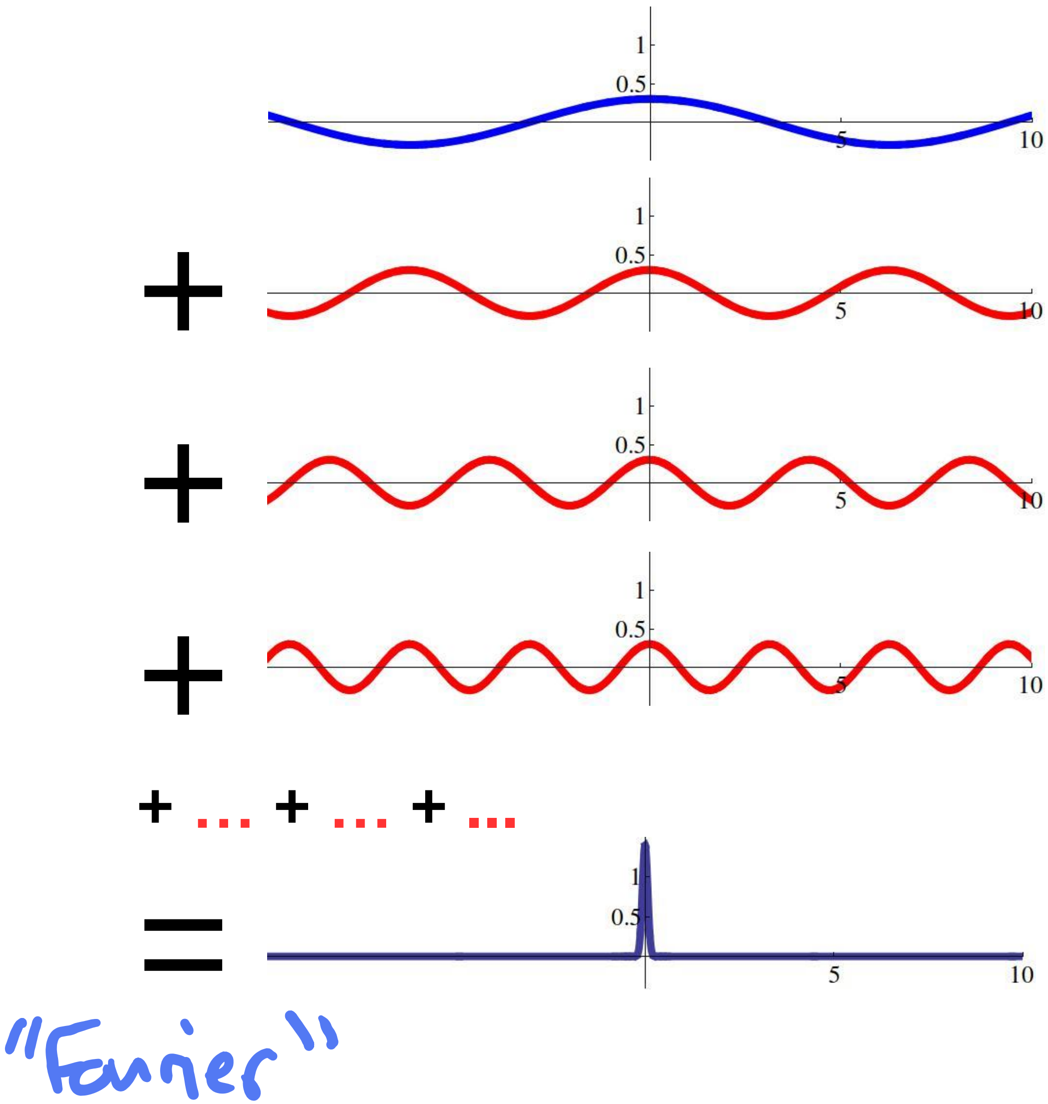
# Ingredients: Spacetime

- Fields can move energy through space in the form of propagating waves
- Waves can be added to cancel each other out… or magnify each other… or localize in space …



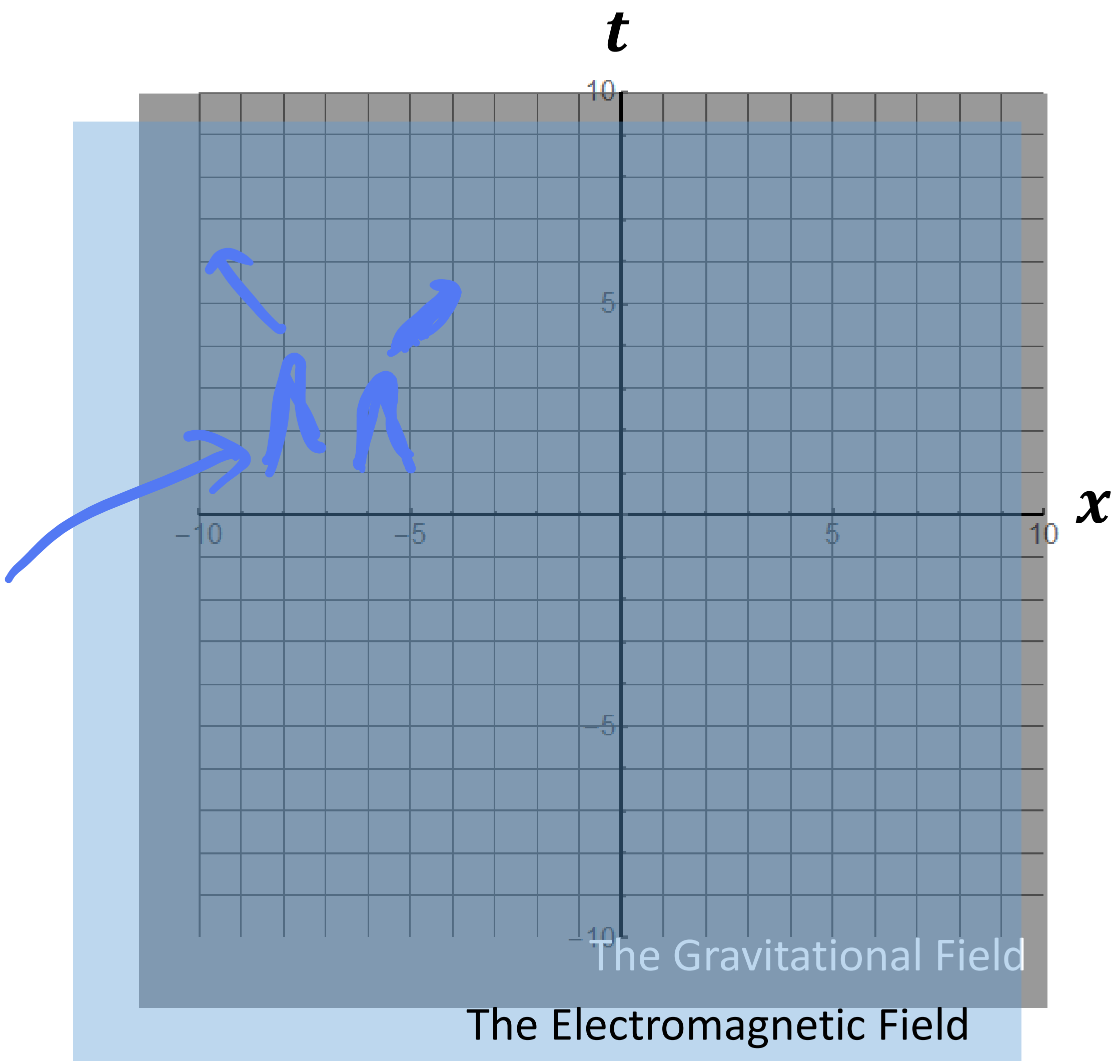
# Ingredients: Spacetime

- Fields can move energy through space in the form of propagating waves
- Waves can be added to cancel each other out… or magnify each other… or localize in space … or become point-like



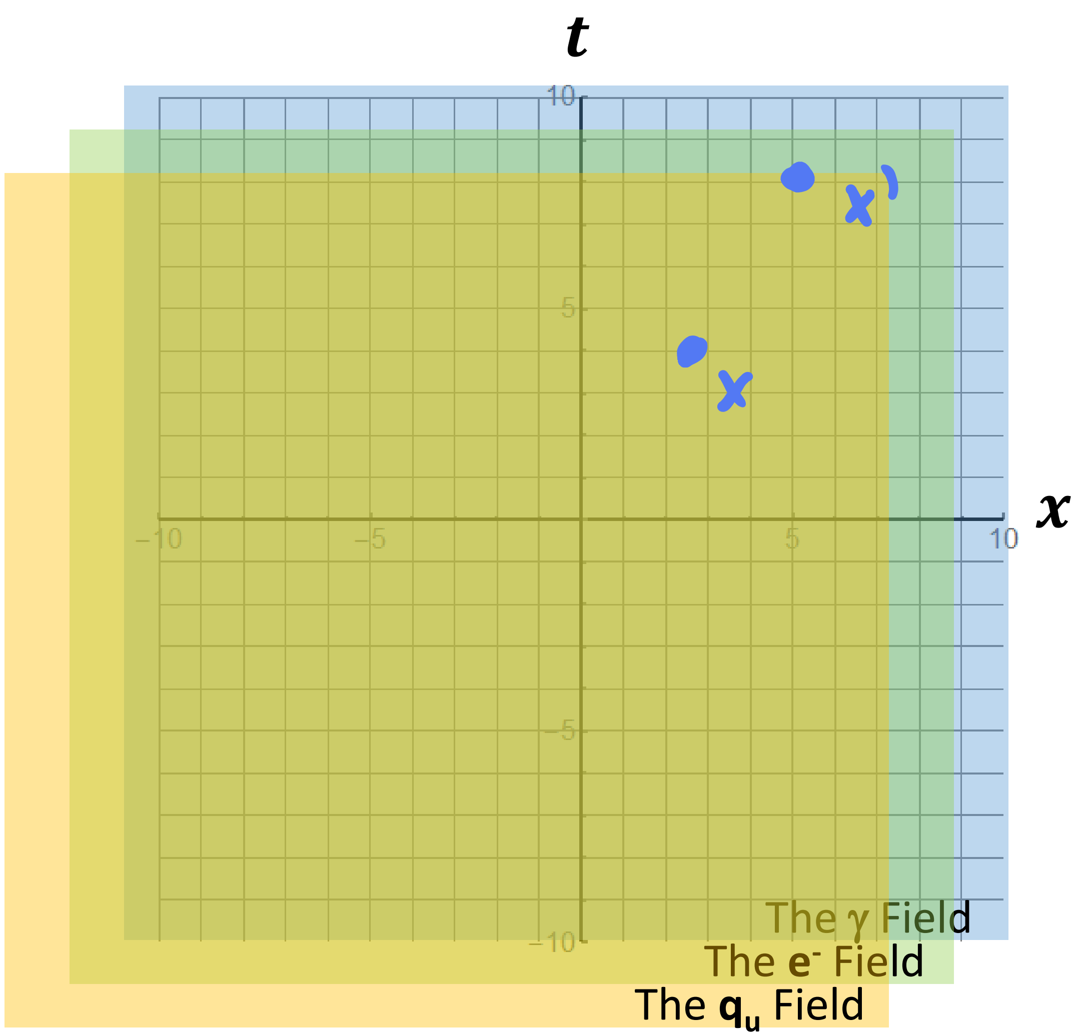
# Ingredients: Spacetime

- Quantum fields support the propagation of localized (possibly point-like) fluctuations in the energy profile of the field



# Ingredients: Spacetime

- Quantum fields support the propagation of localized (possibly point-like) fluctuations in the energy profile of the field
- We can thus view all particles as being excitations in their own quantum field



# Quantum Field Theory in a Nutshell

- *Anything that can happen*, will happen with *some probability*

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- *Anything that can happen*, will happen with *some probability*
- Imagine a toy universe containing two types of particles
  - Particle ***A***
  - Particle ***B***

*Anything that can happen*, will happen with *some probability*

“List of Things that Can Happen”

# Quantum Field Theory in a Nutshell

- *Anything that can happen*, will happen with *some probability*
- Imagine a toy universe containing two types of particles
  - Particle ***A***
  - Particle ***B***
- This one item implies the possibility of three distinct phenomena

*Anything that can happen*, will happen with *some probability*

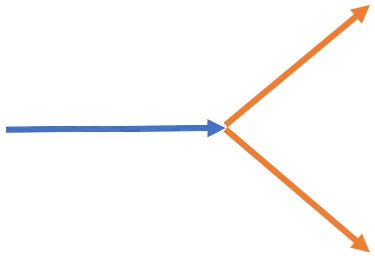
<u>"List of Things that Can Happen"</u>

1) ***ABB***

# Quantum Field Theory in a Nutshell

- This one item implies the possibility of three distinct phenomena

1) *A* -> *B B*



"Decay"

*Anything that can happen*, will happen with *some probability*

"List of Things that Can Happen"

1) ***ABB***

# Quantum Field Theory in a Nutshell

- This one item implies the possibility of three distinct phenomena

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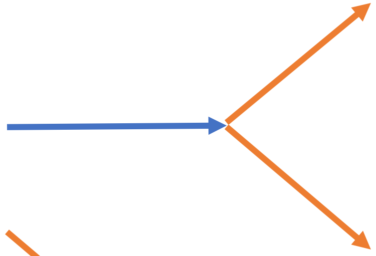
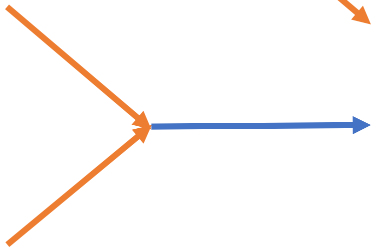


"Decay"

2) *B B* -> *A*



Collider!!

*Anything that can happen*, will happen with *some probability*

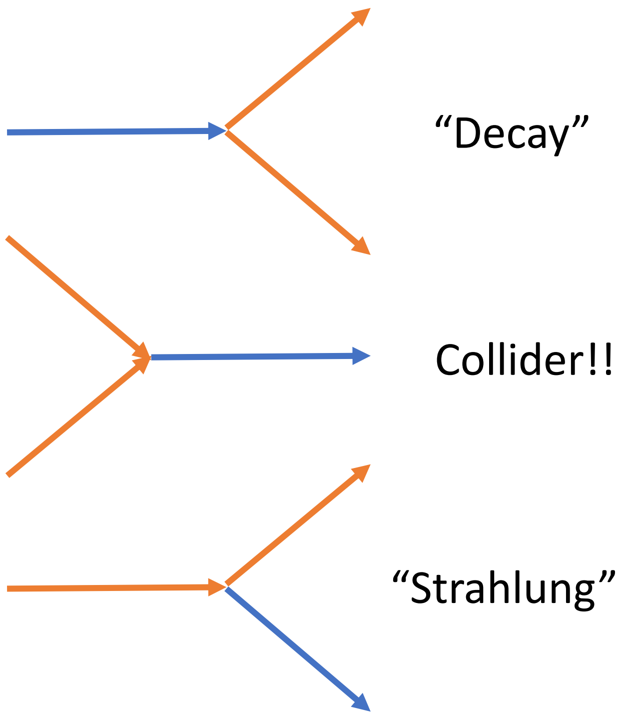
"List of Things that Can Happen"

1) ***ABB***

# Quantum Field Theory in a Nutshell

- This one item implies the possibility of three distinct phenomena

1) ***A* -> *B B***
2) ***B B* -> *A***
3) ***B* -> *B A***




*Anything that can happen*, will happen with *some probability*

"List of Things that Can Happen"

1) ***ABB***

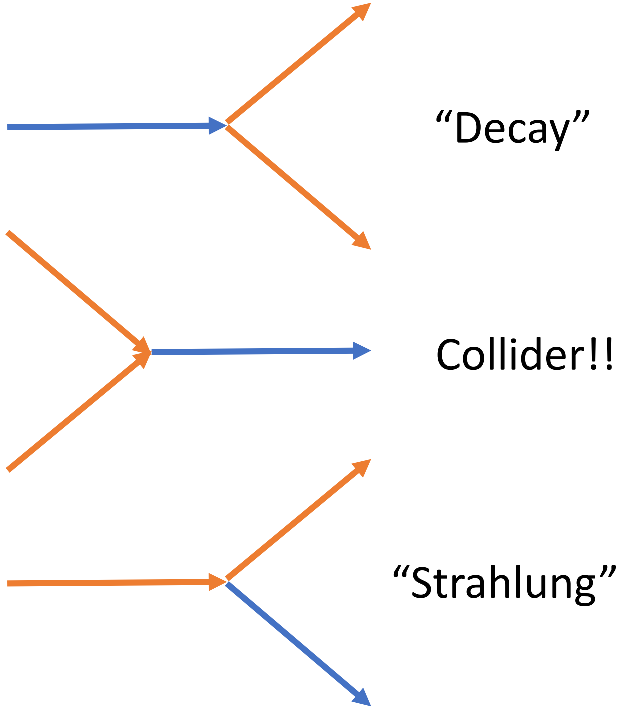
# Quantum Field Theory in a Nutshell

- Diagrams like these are called “Feynman Diagrams”

1) *A* -> *B B*

2) *B B* -> *A*

3) *B* -> *B A*




*Anything that can happen*, will happen with *some probability*

“List of Things that Can Happen”

1) ***ABB***

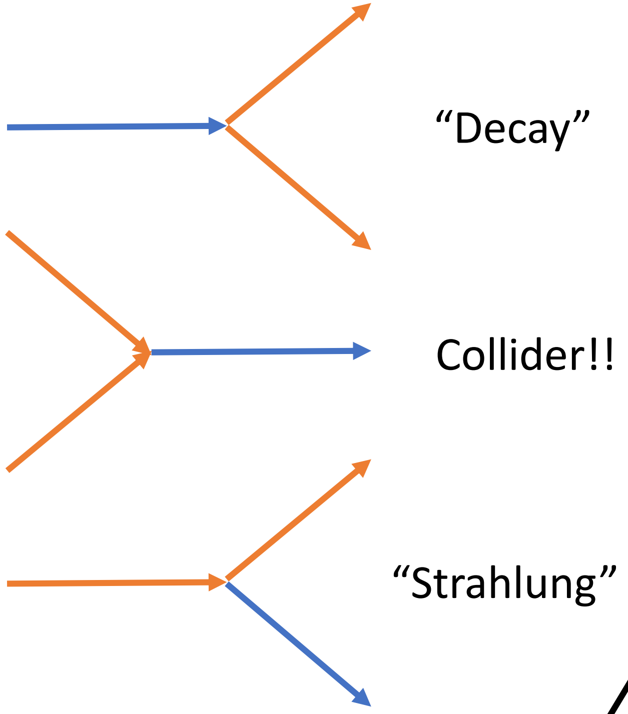
# Quantum Field Theory in a Nutshell

- Diagrams like these are called “Feynman Diagrams”

1) *A* -> ***B B***

2) ***B B*** -> *A*

3) ***B*** -> ***B A***




$\mathcal{L}$ stands for “List” (but really it stands for “Lagrangian”)

*Anything that can happen*, will happen with *some probability*

$\mathcal{L}$ = ***ABB***

“List of Things that Can Happen”

1) ***ABB***

# Quantum Field Theory in a Nutshell

- Diagrams like these are called “Feynman Diagrams”

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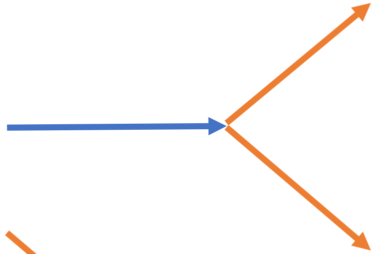
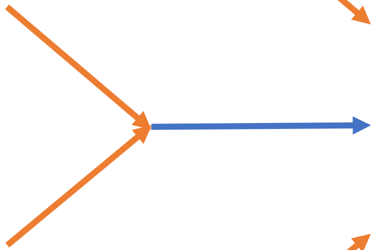


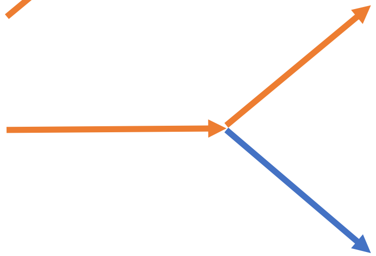
“Decay”

2) *B B* -> *A*



Collider!!

3) *B* -> *B A*



“Strahlung”

*Anything that can happen*, will happen with *some probability*

**This is a “principle”**

**This is a “theory”**

$\mathcal{L} = ABB$

# Infinite Possibilities and Probabilities

- One way to define the "mass" of particle ***A***, is that its inverse squared be proportional to the square root of the **probability of particle *A* propagating from some point $x$ to some other point $x'$**?

$$\frac{1}{m_A^2} \stackrel{\text{def}}{=} \sqrt{\boldsymbol{P(A|x \to x')}}$$

*Anything that can happen*, will happen with *some probability*

$\mathcal{L}$ = ***ABB***

# Infinite Possibilities and Probabilities

- In quantum theory, the square root of the probability (“amplitude”), is equal to the sum of all the independent contributing possibilities.

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- This is analogous to the way probabilities of independent events add in classical probability theory

$\boldsymbol{P}(\mathbf{10})$ = **4/52**

$\boldsymbol{P}(\boldsymbol{KQJ})$ = **12/52**

$\boldsymbol{P}(\mathbf{10}\ ||\ \boldsymbol{KQJ})$ = $\boldsymbol{P}(\mathbf{10})$ + $\boldsymbol{P}(\boldsymbol{KQJ})$

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$$+ \quad ???$$

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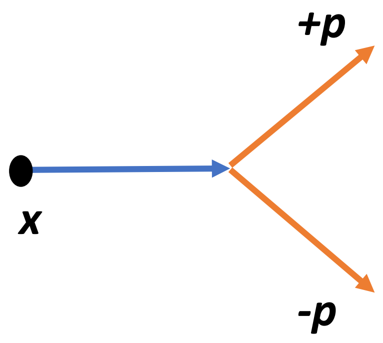


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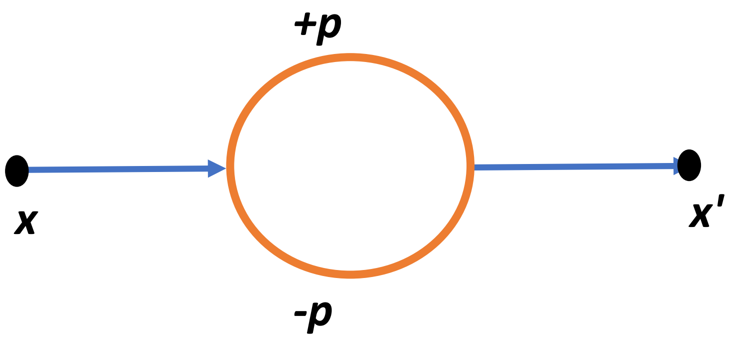


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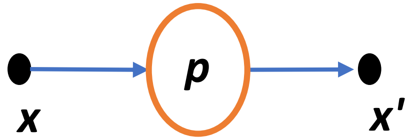
$+$

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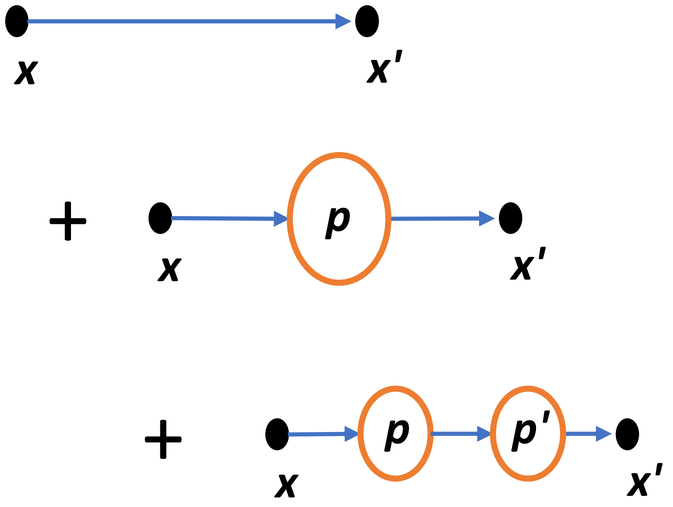


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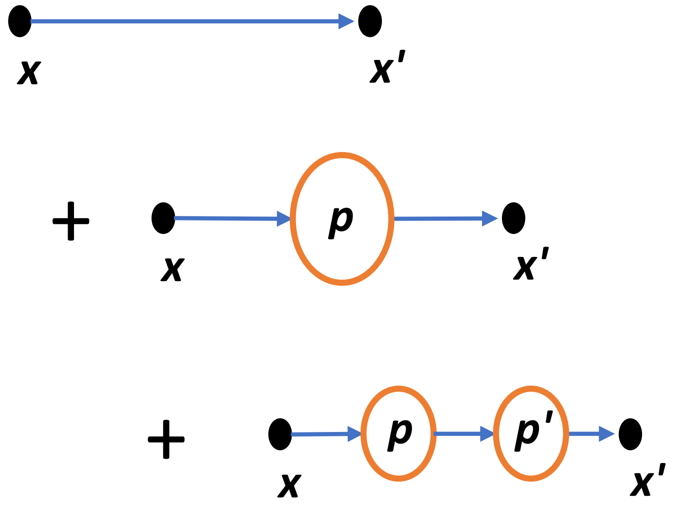


+ ..... = ∞ **FAIL**

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# Infinite Possibilities and Probabilities

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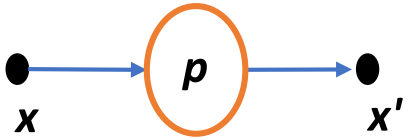
$+$




Each item on this list needs a number ***g*** in [0,1] that parameterizes the “probability” of each item “happening”

*Anything that can happen*, will happen *with some probability*

$\mathcal{L} = gABB$

# Infinite Possibilities and Probabilities

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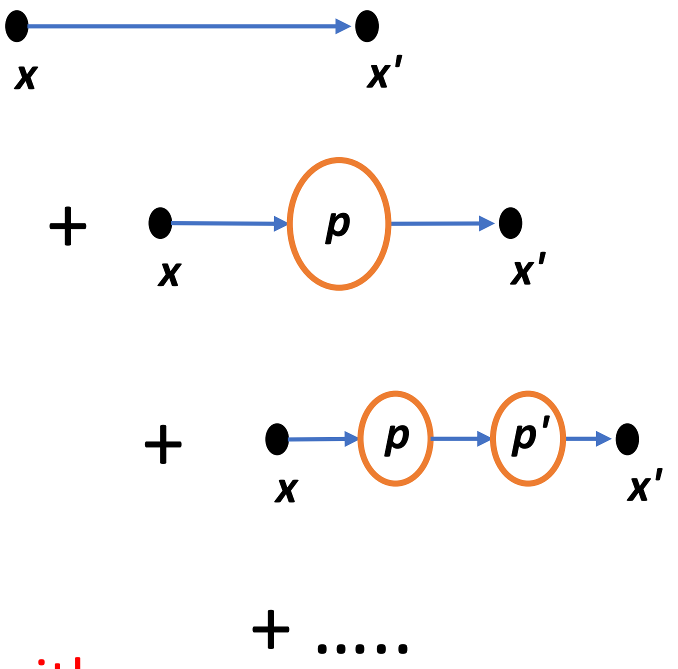


$g^0$

$g^2$

$g^4$

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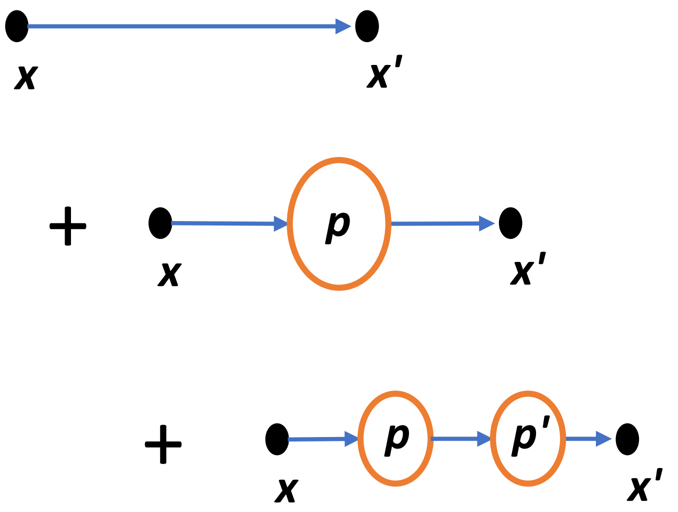


+ .....

$g^0$

$g^2$

$g^4$

$$\mathbf{2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots}$$

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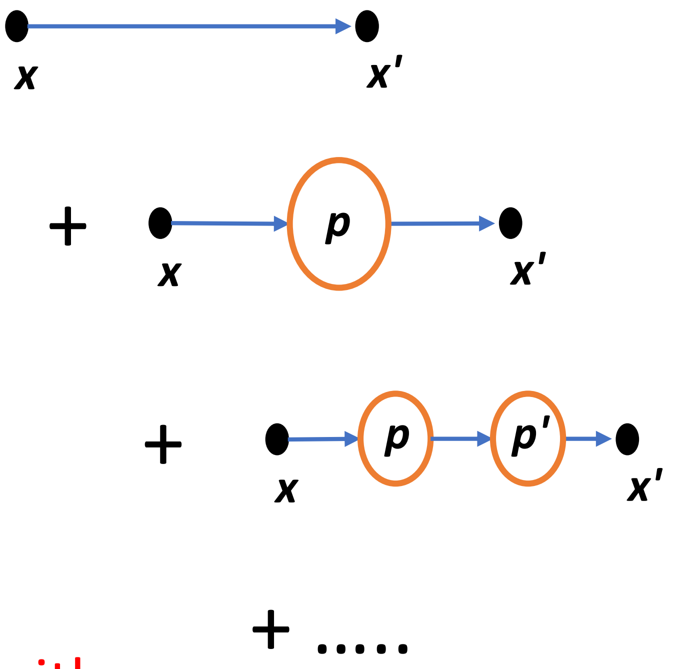


$g^0$

$g^2$

$g^4$

$$\sum_{n=0}^{\infty} g^n = \frac{\mathbf{1}}{\mathbf{1} - \boldsymbol{g}}$$

*Anything that can happen*, will happen *with some probability*

$\mathcal{L} = gABB$

# Quantum Particles: Feynman's Picture

- In quantum theory, the square root of the probability ("amplitude"), is equal to the sum of all the independent contributing possibilities.

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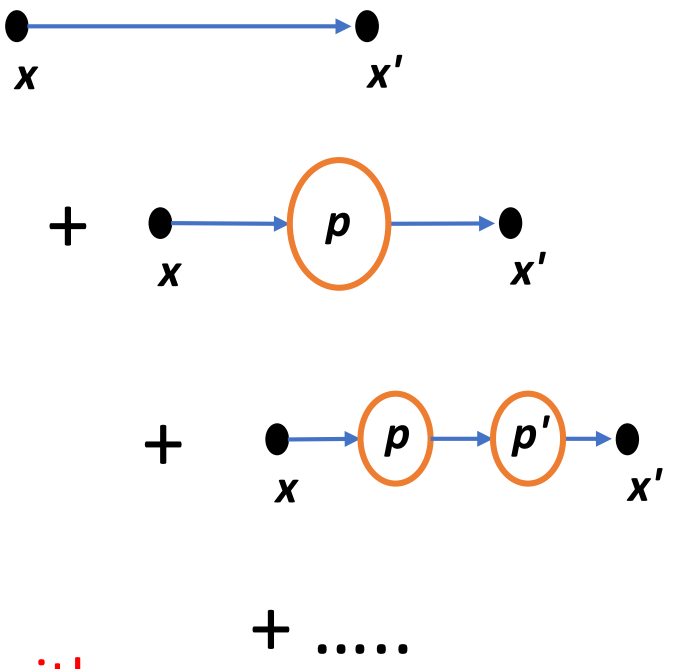


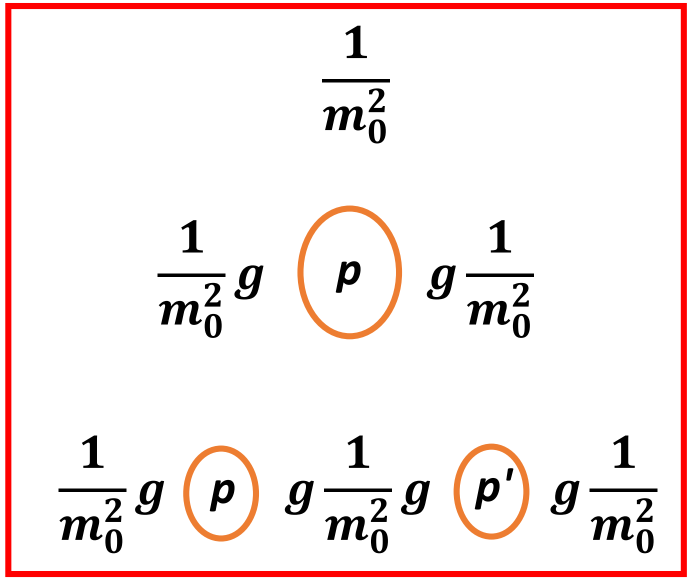


$$\sum_{n=0}^{\infty} g^n = \frac{1}{1-g}$$

*Anything that can happen*, will happen with *some probability*

$\mathcal{L} = gABB$

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$$\frac{\mathbf{1}}{\boldsymbol{m}_0^2}$$

$$\frac{\mathbf{1}}{\boldsymbol{m}_0^2}\boldsymbol{g} \quad \boldsymbol{p} \quad \boldsymbol{g}\frac{\mathbf{1}}{\boldsymbol{m}_0^2}$$

$$\frac{\mathbf{1}}{\boldsymbol{m}_0^2}\boldsymbol{g} \quad \boldsymbol{p} \quad \boldsymbol{g}\frac{\mathbf{1}}{\boldsymbol{m}_0^2}\boldsymbol{g} \quad \boldsymbol{p'} \quad \boldsymbol{g}\frac{\mathbf{1}}{\boldsymbol{m}_0^2}$$

$$\sum_{n=0}^{\infty} g^n = \frac{\mathbf{1}}{\mathbf{1} - \boldsymbol{g}}$$

$$\sum_{n=0}^{\infty} \frac{1}{m_0^2}\left(g\,O\,g\,\frac{1}{m_0^2}\right)^n = \frac{1}{m_0^2 - g^2 O}$$

*Anything that can happen*, will happen *with some probability*

$\mathcal{L} = \boldsymbol{gABB}$

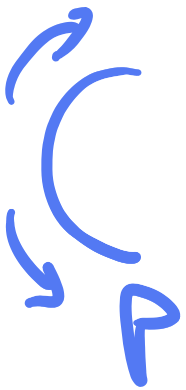
# Quantum Particles: Feynman's Picture

- In quantum theory, the square root of the probability ("amplitude"), is equal to the sum of all the independent contributing possibilities.

$$\frac{1}{m_A^2} \stackrel{\text{def}}{=} \sqrt{P(A|x \to x')} = \frac{1}{m_0^2 - g^2 \textcircled{p}}$$

$$\textcircled{p} = \int_0^\infty dp\, f(p)$$

$$m_A^2 = m_0^2 - g^2 \textcircled{p}$$



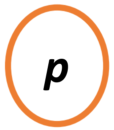

*Anything that can happen*, will happen with *some probability*

$\mathcal{L} = gABB$

# Quantum Particles: Feynman's Picture

- In quantum theory, the square root of the probability ("amplitude"), is equal to the sum of all the independent contributing possibilities.

$$m_A^2 = m_0^2 - g^2 \int_0^\infty dp\, f(p)$$




$$= \int_0^\infty dp\, f(p)$$

*Anything that can happen*, will happen *with some probability*

$$\mathcal{L} = gABB$$

# Quantum Particles: Feynman's Picture

- This formula assumes that this theory does not change when the momentum scale of the theory is very high.

$$m_A^2 = m_0^2 - g^2 \int_0^\infty dp\, f(p)$$

$$(p) = \int_0^\infty dp\, f(p)$$

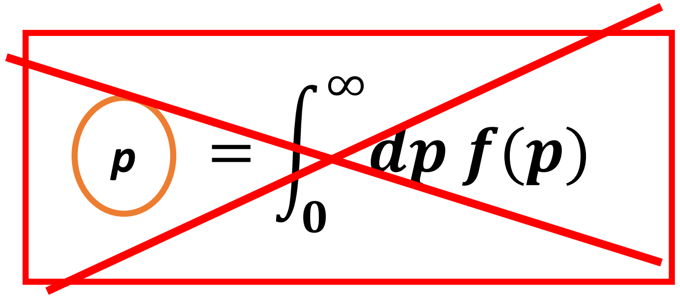
*Anything that can happen*, will happen *with some probability*

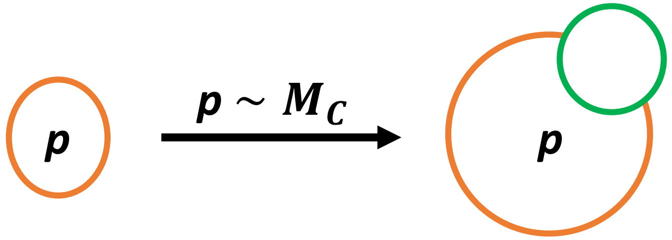
$\mathcal{L} = gABB$

# Quantum Particles: Feynman's Picture

- Suppose there was some new particle ***C*** with a very heavy mass

$$m_A^2 = m_0^2 - g^2 \int_0^\infty dp\, f(p)$$




*Anything that can happen*, will happen *with some probability*

$$\mathcal{L} = gABB + gBCC + M_C CC$$

# Quantum Particles: Feynman's Picture

- Suppose there was some new particle ***C*** with a very heavy mass
- We should define some "cutoff" momentum scale $\boldsymbol{\Lambda}$ at which point we can claim to be agnostic about the "deeper" theory

$$\boldsymbol{m_A^2 = m_0^2 - g^2 \int_0^{\Lambda} dp\, f(p) - g^2 \int_{\Lambda}^{\infty} dp\, f(p)}$$

*Anything that can happen*, will happen with *some probability*

$$\mathcal{L} = \boldsymbol{gABB + gBCC + M_C CC}$$

# Quantum Particles: Feynman's Picture

- Suppose there was some new particle ***C*** with a very heavy mass
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$$\boldsymbol{m_A^2 = m_{IR}^2(\Lambda) + m_{UV}^2\,(\Lambda)}$$

$\Lambda^2$

$p = \frac{h}{\lambda}$

$e^+$ spin $\frac{1}{2}$

$q_u$ spin $\frac{1}{2}$

$$\boldsymbol{m_{IR}^2(\Lambda) = m_0^2 - g^2 \int_0^{\Lambda} dp\, f(p)}$$

$$\boldsymbol{m_{UV}^2(\Lambda) = -g^2 \int_{\Lambda}^{\infty} dp\, f(p)}$$

*Anything that can happen*, will happen *with some probability*

$\mathcal{L} = g\boldsymbol{ABB}$

# The Doctrine of Effective Theories

- For any theory of nature it must be true that $\boldsymbol{m_{IR}^2(\Lambda) \gg m_{UV}^2\,(\Lambda)}$
- *Also known as the principle of scale separation*

$$\boldsymbol{m_A^2 = m_{IR}^2(\Lambda) + m_{UV}^2\,(\Lambda)}$$

$$\boldsymbol{m_{IR}^2(\Lambda) = m_0^2 - \; g^2 \int_0^{\Lambda} dp\, f(p)}$$

$$\boldsymbol{m_{UV}^2(\Lambda) = -g^2 \int_{\Lambda}^{\infty} dp\, f(p)}$$

*Anything that can happen*, will happen with *some probability*

$\mathcal{L}$ = ***gABB***