Modeling Quantum Waves and Probability

Jack Carlton
Quarknet
Hereford High School

August 5th, 2016

Abstract

Over the course of six weeks, I researched established wave function models. These models have already made large impacts on technology today. I used a couple of simulations in order to "look into" the quantum world. As a result, I am now able to comprehend the nature of particles and their waveforms.

Introduction

Before knowledge of the wave-particle duality, not much was known about the behavior of fundamental particles such as electrons. Now, we can model the behavior of such particles and use them to our benefit. So far, scientists and engineers have only scratched the surface of quantum mechanical applications.

A representational visualization of wave-particle duality

[6]

Wavefunctions

Wave function (ψ) of position and time: $\psi = \psi(x,t) = f(t)\psi(x)$

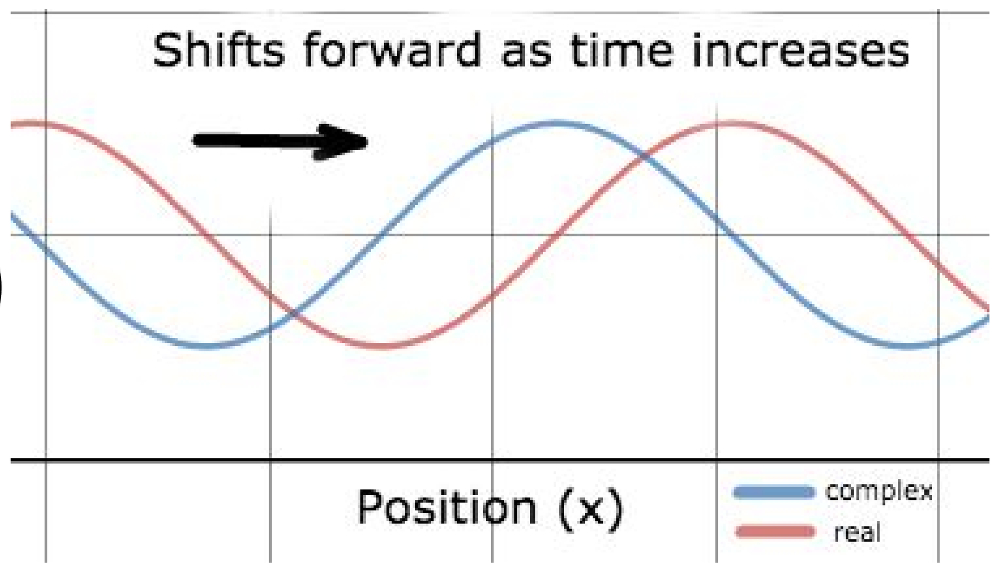
Shrödinger Equation:

$$i\hbar\frac{\partial\psi}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U\psi = E\psi$$ [1]

Time evolution | Hamiltonian operator (H) for position | Quantum energy level

For a particle in free space, the wavefunction is of the form:

$$\psi = Ae^{i(kx-wt)}$$



where:
k = wave number = $\frac{2\pi}{\lambda} = \frac{\sqrt{2mE}}{\hbar}$
ω = angular frequency = $\frac{E}{\hbar}$

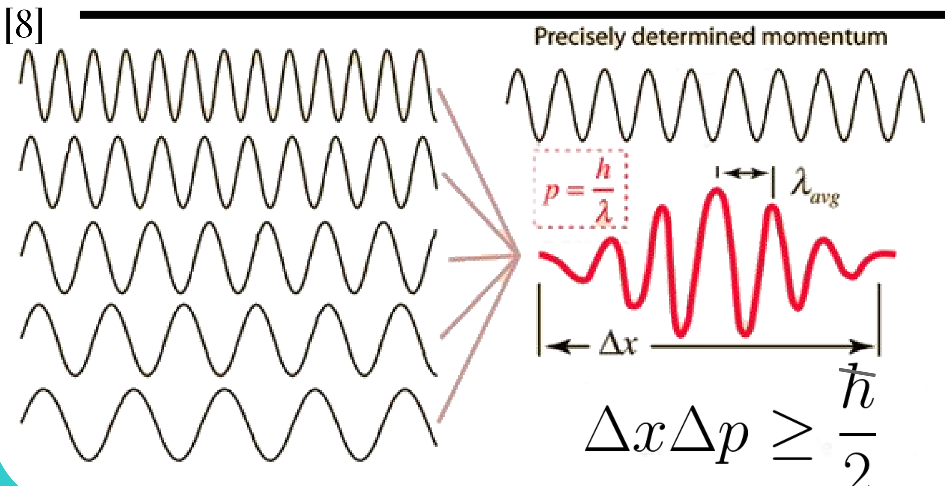
A wave may be also superposed between multiple energy values. It then takes the form:

$$\psi = \sqrt{P_1}\psi_1 + \sqrt{P_2}\psi_2 + \ldots + \sqrt{P_n}\psi_n + \ldots$$ [5]

Where P is the probability of finding the wave at that energy value such that: $\sum_{n=1}^{n} P_n = 1$

Uncertainty Principle

[8]



$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

One cannot expect to know position and momentum with unreasonable precision at the same time[2]. This is a feature of waves.

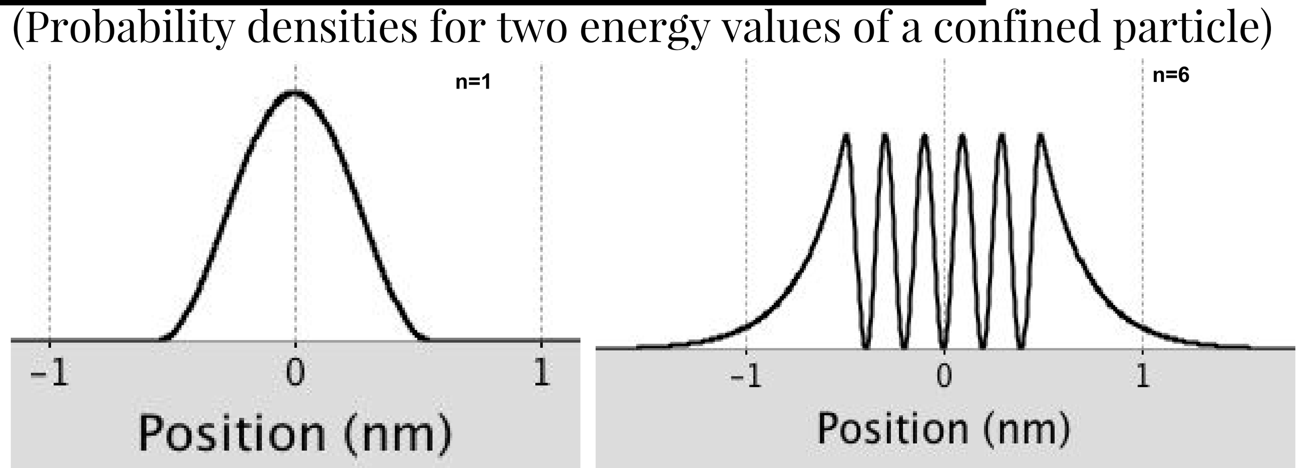
Probability Density

For any wave function:

$$P = \overline{\psi}\psi = |\psi|^2$$

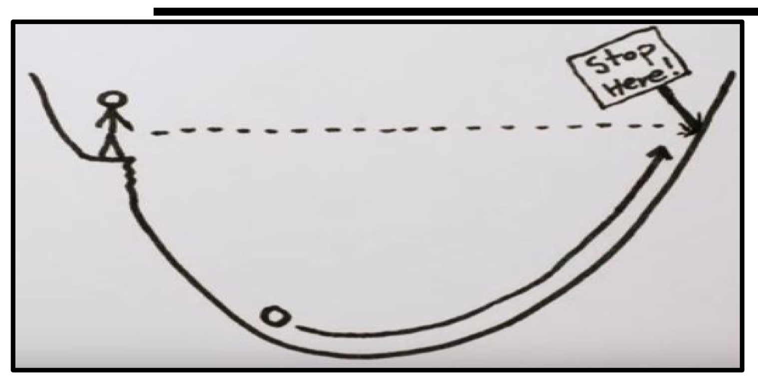
$$\int_{-\infty}^{\infty} |\psi|^2 = 1$$

(Probability densities for two energy values of a confined particle)



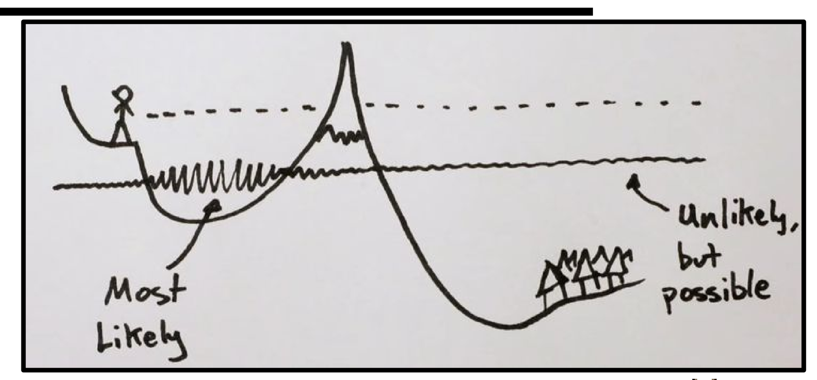
Time doesn't affect probability density unless the function is superposed.

Quantum Tunneling

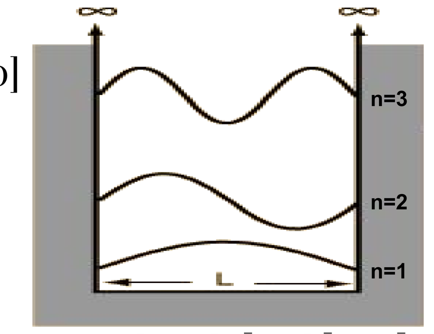


[9]

[9]

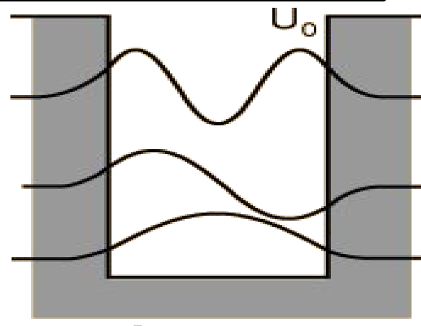


[10]



Particle is stuck!

Particle can pass through!



[10]

A wave inside the box (i.e. free space) has a function (ψ):

$$\psi_a = Asin(kx)f(t)$$

where:
A = Amplitude = $\sqrt{\frac{2}{L}}$
k = wave number = $\frac{\pi n}{L}$
f(t) = time evolution = $e^{-i\omega t}$

A wave through a barrier has a function (ψ) and tunneling probability (T):

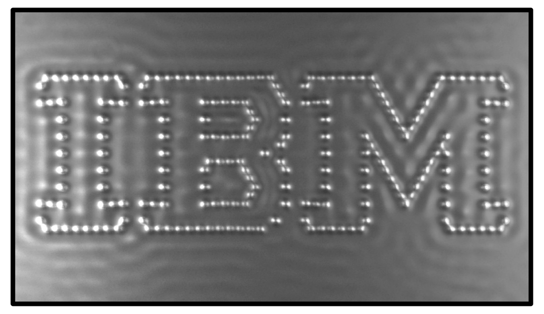
$$\psi_b = Ce^{-\alpha x}f(t);\, T = e^{-2\alpha L}$$

where L is the length of the *barrier* and [4]

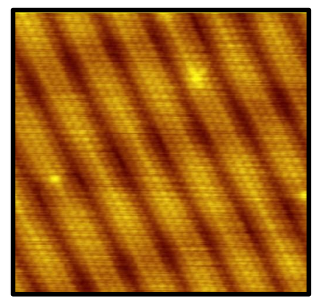
$$\alpha = \frac{\sqrt{2m(U_0 - E)}}{\hbar^2}$$ (from Shrödinger's equation)

Scanning Tunneling Microscope (STM)

IBM written in atoms (top) and the surface of gold (bottom) using STM



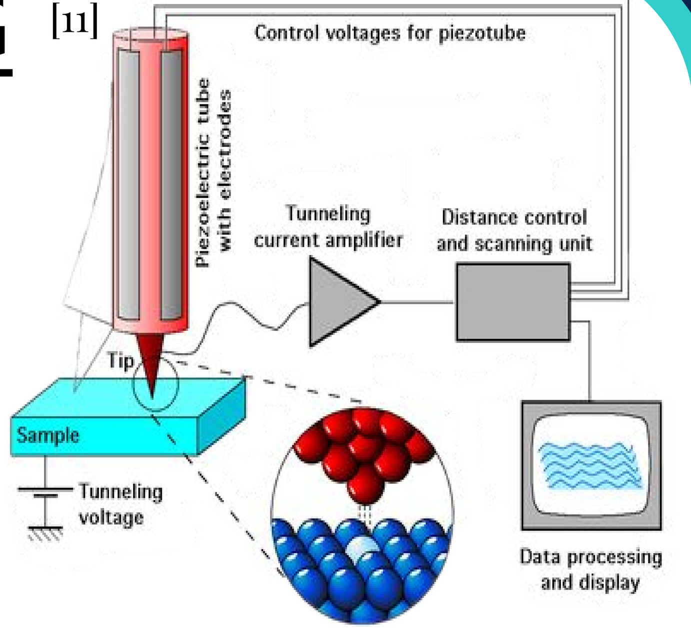
[14]



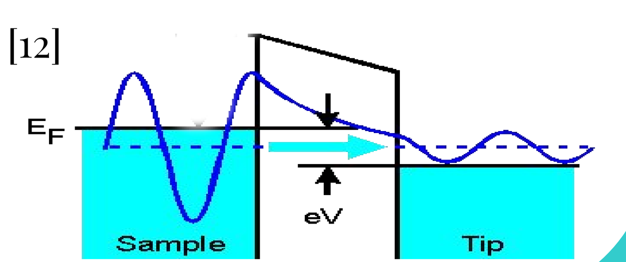
[13]

The STM uses quantum tunneling across a thin vacuum to locate atoms and even their electron orbitals.[3] These are the smallest resolution microscopes in the world.[7]

[11]



[12]



References and Acknowledgements

1. Schrodinger, Erwin (1926). "Quantisierung als Eigenwertproblem". *Annalen der Physik* **384** (4): 273–376.
2. Heisenberg, W. (1927), "Über den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik", *Zeitschrift für Physik*(in German) **43** (3–4): 172–198
3. Binnig, G.; Rohrer, H. (1986). "Scanning tunneling microscopy".*IBM Journal of Research and Development*. **30** (4): 355–69.
4. Gurney, R. W.; Condon, E. U. (1928). "Quantum Mechanics and Radioactive Disintegration". *Nature*. **122** (3073): 439.
5. Bohr, N. (1927/1928). "The Quantum Postulate and Recent Development of Atomic Theory". *Nature* Supplement 14 April 1928, **121**: 580–590
6. Kurzgesagt. "Quantum Computers Explained" [Image]. 08 Dec. 2015. http://www.youtube.com
7. "Applications of tunneling" Archived 23 July 2011 at the Wayback Machine.. Simon Connell 2006.
8. "Uncertainty Principle" [Image]. (n.d.). Retrieved from http://hyperphysics.phy-astr.gsu.edu/
9. MinutePhysics. "Quantum Tunneling Analogies" [Image]. 14 August 2011. Retrieved from http://www.youtube.com
10. "Quantum Tunneling Wells" [Image]. (n.d.). Retrieved from http://www.hyperphysics.phy-astr.gsu.edu
11. Schmid, Michael. "STM diagram" [Image]. (n.d.). 7 June 2005. Retrieved from https://en.wikipedia.org/wiki/Scanning_tunneling_microscope
12. "STM tunneling" [Image]. (n.d.). Retrieved from http://www2.fkf.mpg.de/ga/research/stmtutor/stmtheo.html
13. "Gold surface" [Image]. (n.d.). Retrieved from https://en.wikipedia.org/wiki/Scanning_tunneling_microscope
14. "IBM atoms" [Image]. (n.d.). Retrieved from http://www.extremetech.com/extreme/

Acknowledgements:
- Dr. Morris Swartz
- Jeremy Smith
- Kevin Martz
- Quarknet
- National Science Foundation
- Johns Hopkins University Physics Department