Muons

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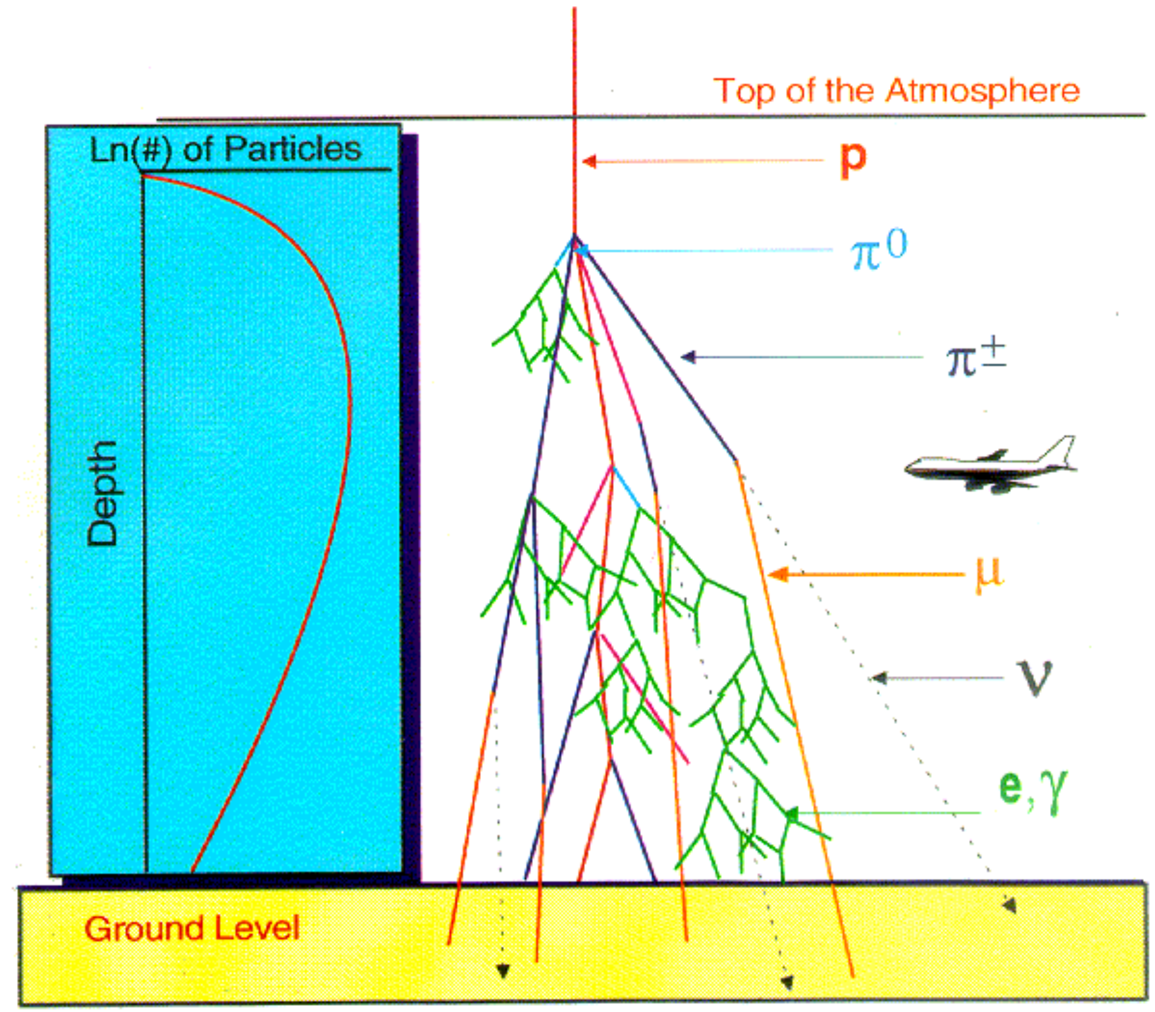
Muons: everything you've ever wanted to know

The muon was first observed in cosmic ray tracks in a cloud chamber by Carl Anderson and Seth Neddermeyer in 1937. It was eventually shown be a heavier brother of the electron:

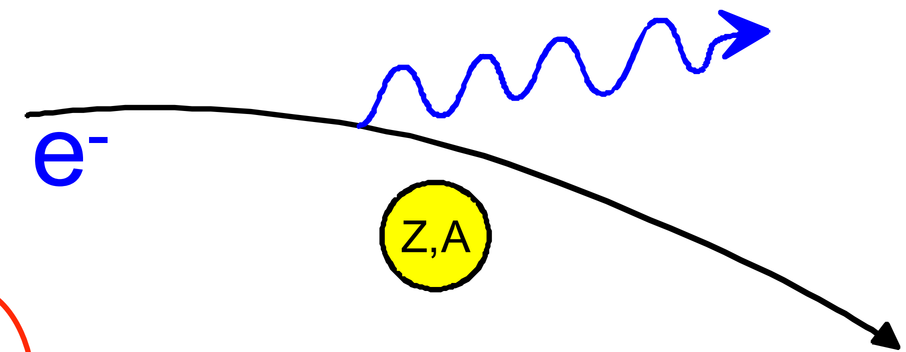
- mass: m_μ=0.1056583715(35) GeV
  - ~200 times heavier than the e
- long lived: $\tau_\mu \sim 2\ \mu s$
  - 10 GeV μ travels on average $\gamma v \tau_\mu$~62km [can transit the atmosphere]
- participates in weak and electromagnetic interactions
  - does not interact strongly with atomic nuclei
  - does emit bremsstrahlung in matter
  - does ionize traversed matter



Extensive Air Showers

Bremsstrahlung

Bremsstrahlung arises if particles
are accelerated in Coulomb field of nucleus



$$\frac{dE}{dx} = 4\alpha N_A \frac{z^2 Z^2}{A}\left(\frac{1}{4\pi\epsilon_0}\frac{e^2}{mc^2}\right)^2 E \ln\frac{183}{Z^{\frac{1}{3}}} \propto \frac{E}{m^2}$$

i.e. energy loss proportional to $1/m^2$ ➙ main relevance for electrons ...

... or ultra-relativistic muons

Consider electrons:

$$\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 \cdot E \ln\frac{183}{Z^{\frac{1}{3}}}$$

$$\frac{dE}{dx} = \frac{E}{X_0} \quad \text{with} \quad X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln\frac{183}{Z^{\frac{1}{3}}}}$$

[Radiation length in g/cm²]

➙ $E = E_0 e^{-x/X_0}$

After passage of one X_0 electron has lost all but $(1/e)^{th}$ of its energy

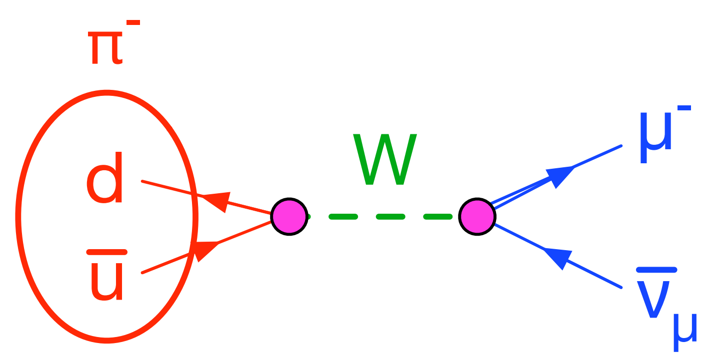
[i.e. 63%]

An electron traversing water loses 1/e of it's energy to photon radiation in 0.36m. A muon loses 1/e of it's energy to photon radiation in 15.5 km of water.

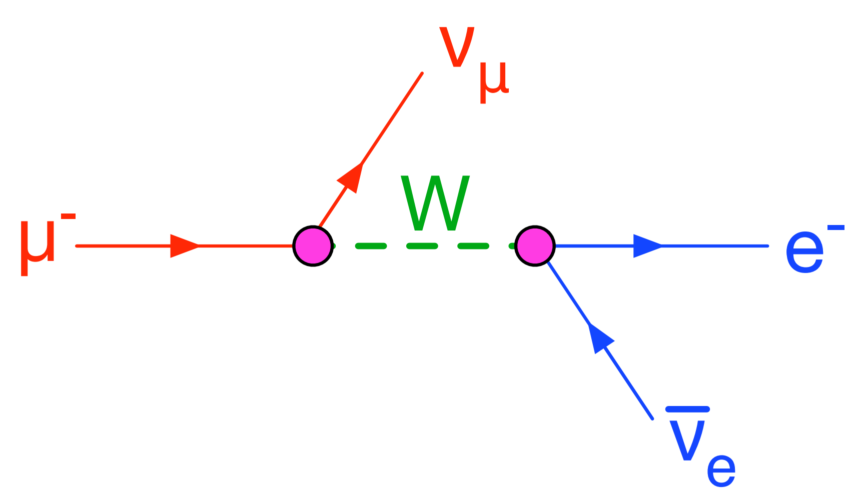
The 10 GeV muon loses all of its energy to ionization in about 50m of water.

- muons are very penetrating particles
- there is some reasonable probability that low energy muons range out [stop] in ordinary material from ionization energy loss
  - stopped μ^- can replace e^- in atoms: Bohr rad ($r_B = [\alpha m c]^{-1}$) is 200 times smaller
  - μ^- can be captured by nuclear protons: $\mu^- + p \rightarrow n + \nu_\mu$
  - stopped μ^+ decays without atomic binding
- production in cosmic ray showers is dominantly via π-decay
  - $\mu^+ \mu^-$ pair production is also possible from energetic γ rays

Most cosmic ray muons are produced by the weak decays of charged pions $\pi^{\pm} \to \mu^{\pm} + \nu_{\mu}$. The pions are the lightest strongly interacting particles, therefore the muon cannot decay to strongly interacting particles [which are all heavier],



The muon decays only weakly [because it's charged and a photon mediated decay can't happen],



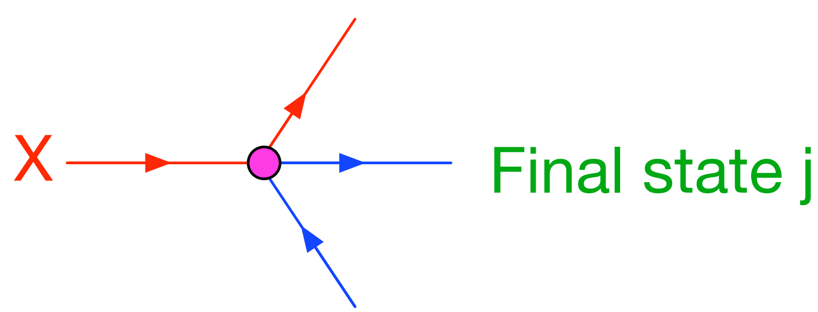
The decay $\mu \to e\nu_e\nu_\mu$ is the dominant one [other decays happen at higher order and are quite rare].

Particle Decays

Particles are not born with little clocks inside them. For any given infinitesmal of time dt, they have a constant probability of decaying. This can be characterized by considering the fraction of a population N that decay,

$$\frac{dN}{N} = -\Gamma_t dt \quad \rightarrow \quad N = N_0 e^{-\Gamma_t t}$$

Where Γ_t is the total decay rate of the particle X and N_0 is the number of particles at time $t=0$. If X decays into several final states [labelled with an index j], then Γ_t is the sum of the partial decay rates into each final state Γ_j,



$$\Gamma_t = \sum_j \Gamma_j$$

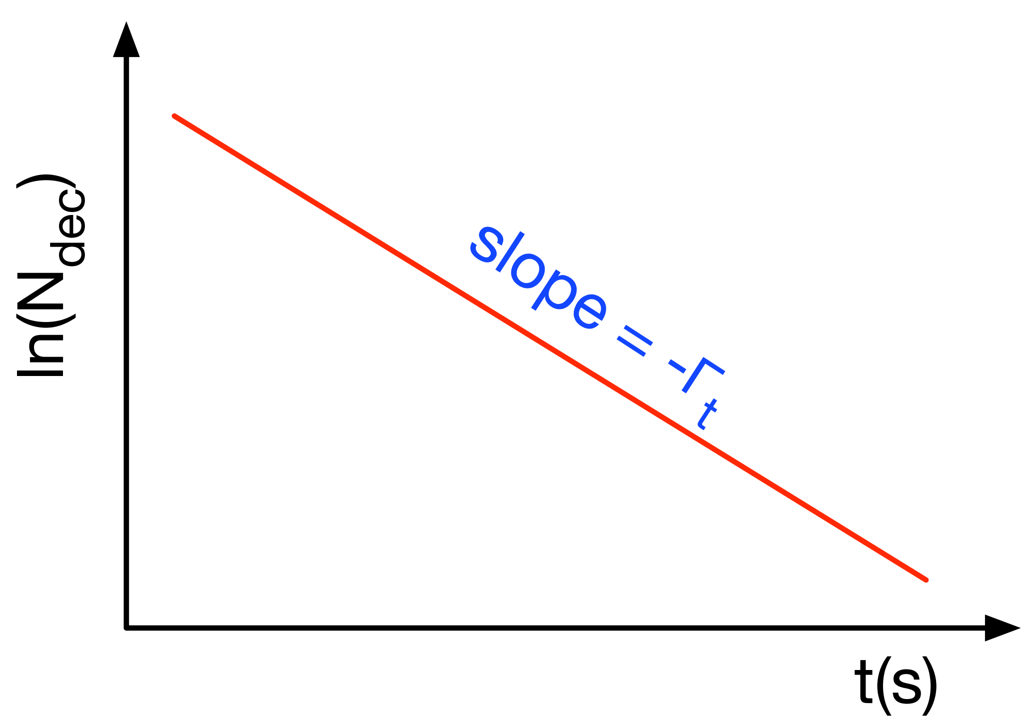
Note that we can define the branching fraction of X->j in terms of the decay rates,

$$B(X \rightarrow j) = \frac{\Gamma_j}{\Gamma_t}$$

The total decay rate, Γ_t, is the inverse of the particle lifetime $\tau = 1/\Gamma_t$. Conversion from decay width (energy units) to inverse time units is given by the uncertainty principle and Planck's constant,

$$\Gamma_t(\mathrm{s}^{-1}) = \frac{\Gamma_t(\mathrm{GeV})}{\hbar} = \frac{\Gamma_t(\mathrm{GeV})}{6.582119514(40) \times 10^{-25}\mathrm{GeV}\cdot\mathrm{s}}$$

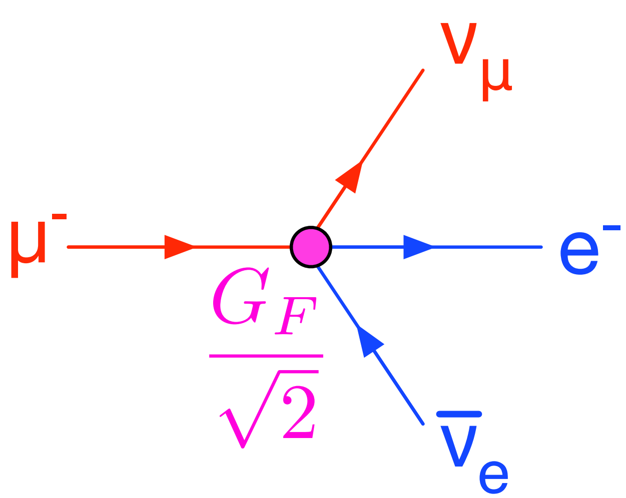
If we plot the number of observed decays vs the time from the stopping of the muon, we expect an exponential distribution. If we plot $\ln(N_{dec})$ vs time, it should be linear with a slope $-\Gamma_t$



If our μ hasn't decayed after τ_μ sec, how much longer will it live on average?

Muon Lifetime

Fermi created a theory of beta decay [weak interactions] in 1933 after Pauli's neutrino hypothesis was publicly presented. It was modified in the later 1950s to include parity violation and works quite well at low energies. It assumes that weak interactions happen at 4 fermion vertex with an interaction strength given by a constant $G_F/2^{1/2}$

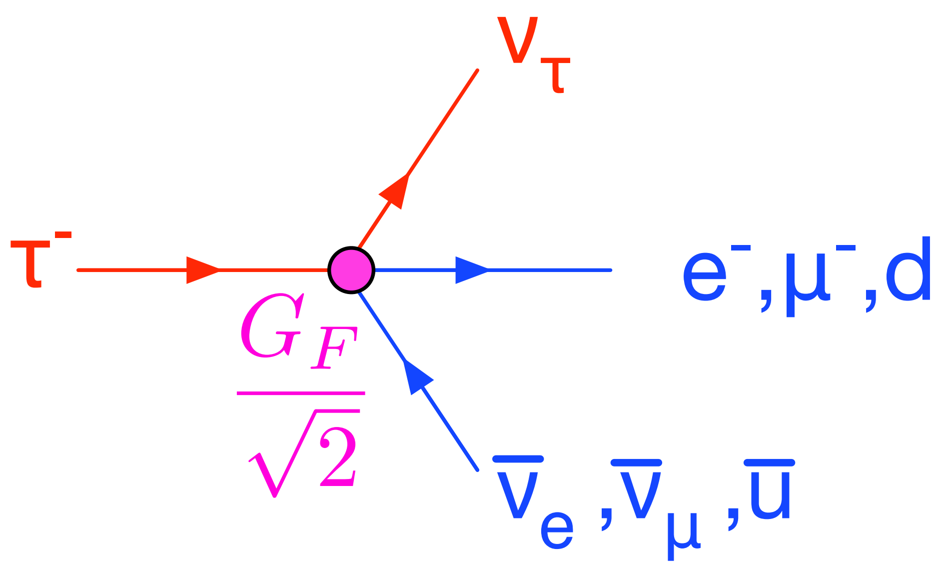


It is straightforward to use this theory to calculate the muon decay rate

$$\Gamma_\mu = \frac{1}{\hbar} \frac{G_F^2 m_\mu^5}{192\pi^3}$$

- G_F has the dimensions of $[\text{energy}]^{-2}$
- assumes that the final state particle masses are much smaller than the muon mass
- the decay rate increases as the 5th power of the muon mass!
  - this expression can be used to calculate the lifetimes of other low energy weak decays ... heavier particles decay much faster!

Another example of this result is the lifetime of the even heavier sibling, the tau lepton, τ



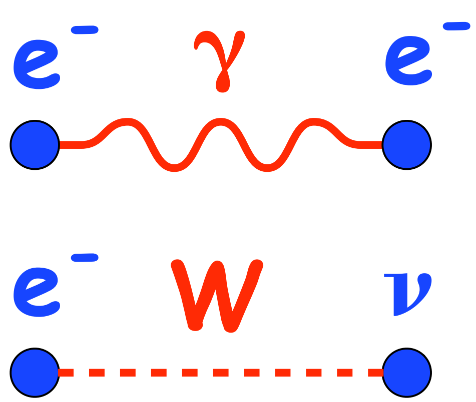
We can modify the last expression to get a pretty good estimate of the tau lepton lifetime and the leptonic branching fractions

$$\Gamma_\tau = \frac{1}{\hbar} \frac{G_F^2 m_\tau^5}{192\pi^3} [1 + 1 + 3]$$

- Where is the factor of 3 coming from?
- What is the approximate branching fraction of $\tau \to \mu \nu_\tau \nu_\mu$?
- The tau mass is 1.777 GeV, how long does it live [if the μ lives 2.2μs] ?

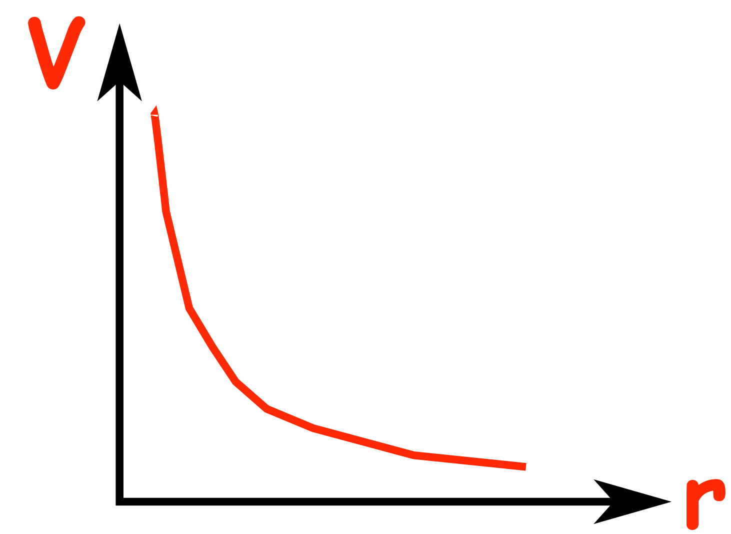
Spontaneous Symmetry Breaking

The architects of the Standard Model needed gauge bosons to transmit the weak force: they are naturally massless (like the photon)



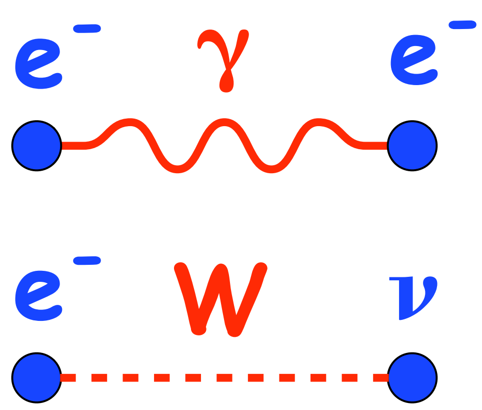
$$V_{ee}(r) = k\frac{e^2}{r}$$

$$V_{e\nu}(r) = k'\frac{g^2}{r}$$



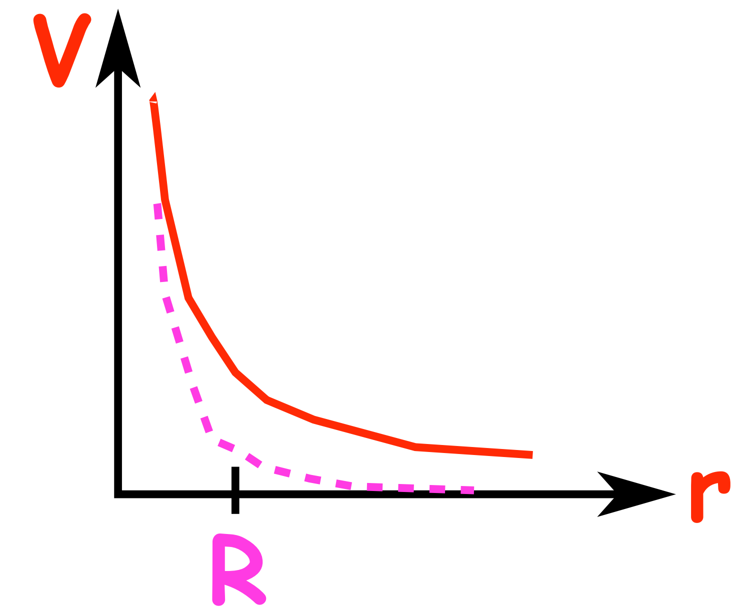
- photon and W boson are massless
  - both electromagnetic and weak forces have infinite range: the potential energy between two charges falls as 1/r
- weak charge g ~ 2e, weak force is stronger than e/m force!
  - describes a totally different universe (fast burning stars, dangerous neutrino radiation, no human life)

They knew that the weak force gauge bosons would have to be massive:



$$V_{ee}(r) = k\frac{e^2}{r}$$

$$V_{e\nu}(r) = k'\frac{g^2}{r}\cdot e^{-\frac{r}{R}}$$



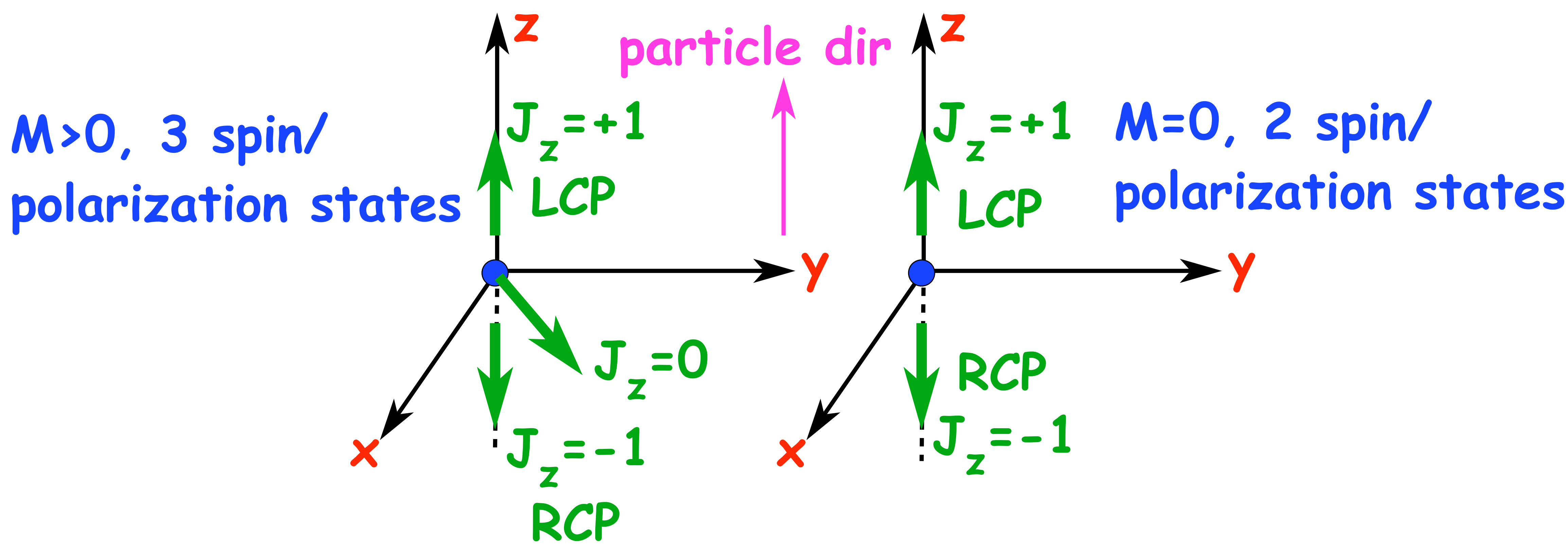
- potential function now acquires an exponentially-short range R

$$R = \frac{\hbar}{Mc}$$

  - short range implies that interaction occurs only when particles approach within distance R: makes interaction effectively weaker
  - range is inversely proportional to gauge boson mass M:
    - ✳ R ~ 1×10^{-15}m (r_N) requires M ~ 100 MeV/c^2 (approx m_π)
    - ✳ R ~ 1×10^{-18}m needed to get correct strength of weak interaction requires M ~ 100 GeV/c^2

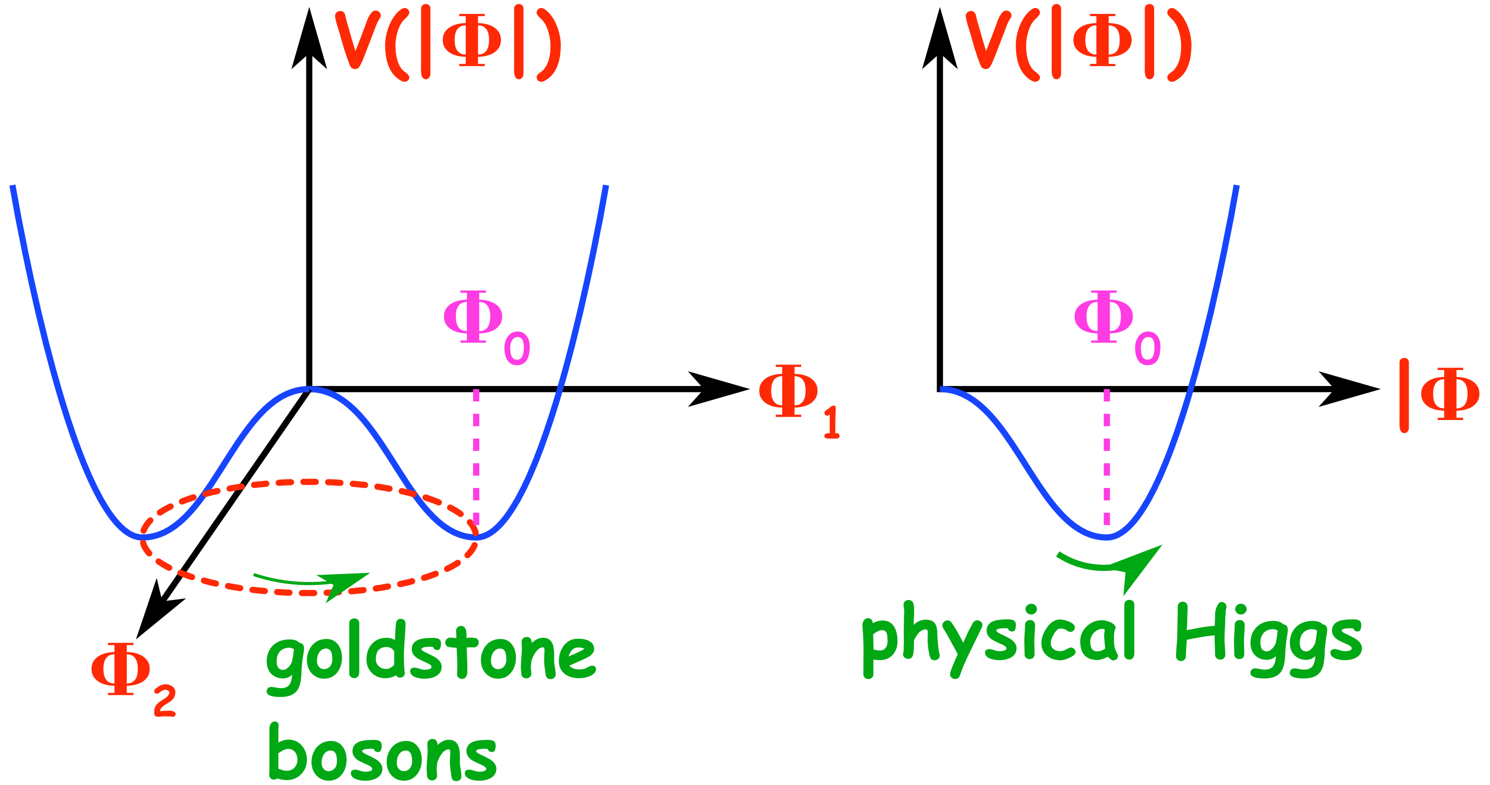
This was a quandry: gauge theories like Quantum Electrodynamics involve massless spin 1 particles (photons) to transmit the force, but any description of weak interactions needed massive spin 1 particles (W,Z).

Actually it's even worse ... massive spin 1 particles have an extra polarization



- massless gauge bosons have left- and right-circular polarizations ($J_z=\pm1$)
- massive gauge bosons also have longitudinal polarization states ($J_z=0$)
  - need to add more quantum fields to theory

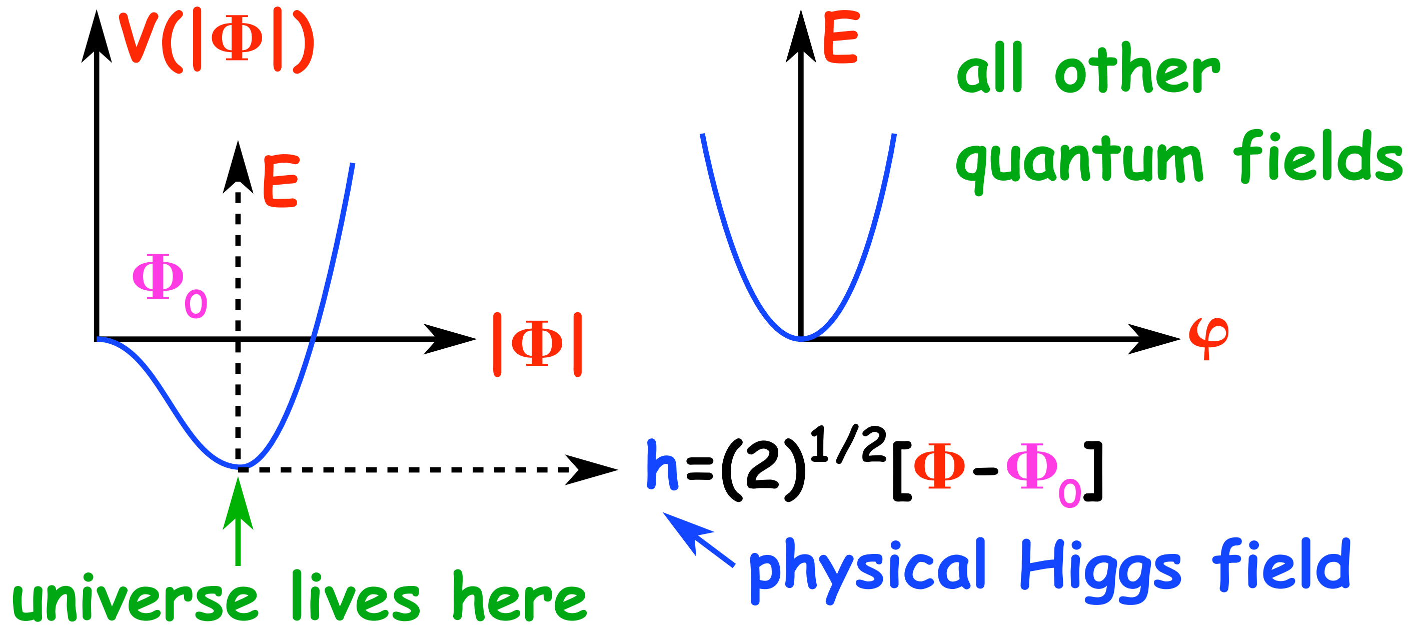
What did they do? They added 4 spin 0 particles (+,-,0,0 charges) and a very unusual potential energy function



$$\Phi_0 = \frac{v}{\sqrt{2}}$$

- universe sits in the bottom of the asymmetric well with non-zero value Φ_0 of the field (called the vacuum expectation value of the Higgs)
  - asymmetric ground state doesn't respect symmetry of theory (SSB)
- 3 states corresponding to motion in the bottom of the well become the longitudinal polarization states of $W^{\pm}$ and Z^0 giving them masses
- oscillations of field transverse to the well correspond to a massive spin 0 particle (Higgs boson)

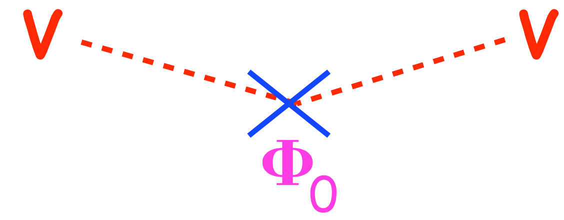
This asymmetry makes the Higgs field unique as compared to all others which have zero average value

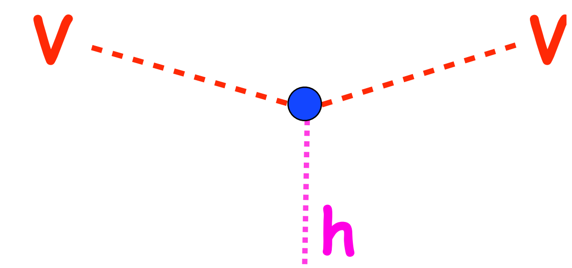


$$\Phi_0 = \frac{v}{\sqrt{2}}$$

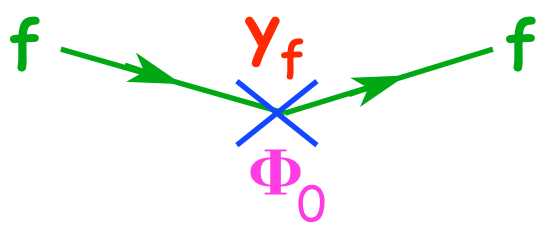
- The Higgs field is the "new ether" that fills the entire universe
  - gives mass to all elementary particles
  - has nothing to do with light propagation
  - does not violate special relativity (like the "old ether")
    - spin 0 fields look the same to all inertial observers

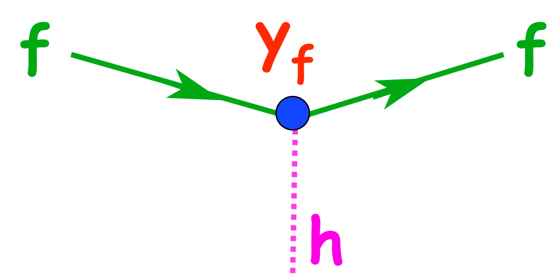
The higgs was "designed" to couple to (and give mass to) the vector bosons.





The vacuum expectation value, Φ_0 =174 GeV, can also give mass to all of the spin 1/2 fermions (quarks and leptons) by coupling them to the Higgs field,





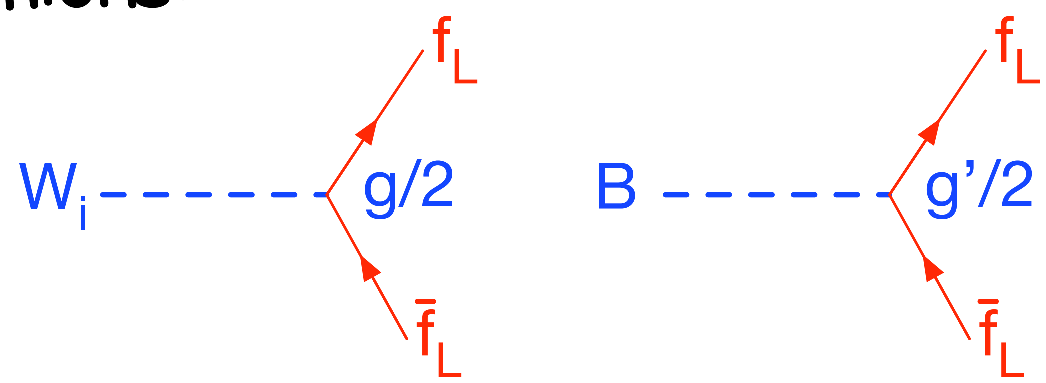
The coupling strength of each fermion to the Higgs is described by a dimensionless Yukawa coupling y_f

- mass of each state is just $m_f = y_f \Phi_0$
- mass of t-quark is "natural"
  - t-quark may play a role in SSB?
- neutrino mass mechanism is probably more complicated

State	Mass (GeV)	Y_f
top	173.2±0.9	0.995±0.005
bottom	4.75	0.027
charm	1.5	0.009
strange	0.1	6×10^{-4}
down	0.005	3×10^{-5}
up	0.002	1×10^{-5}
tau	1.78	0.01
muon	0.106	6×10^{-4}
electron	0.0005	3×10^{-6}
neutrino	$<1\times10^{-10}$	$<6\times10^{-13}$

The Electroweak Standard Model

The theory contains 3 SU(2) gauge bosons [W_1,W_2,W_3] that couple to left-handed fermion doublets and one U(1) guage boson [B] that couples to left- and right-handed fermions:



Where g, g' are dimensionless coupling constants. The physical gauge bosons are linear combinations of these

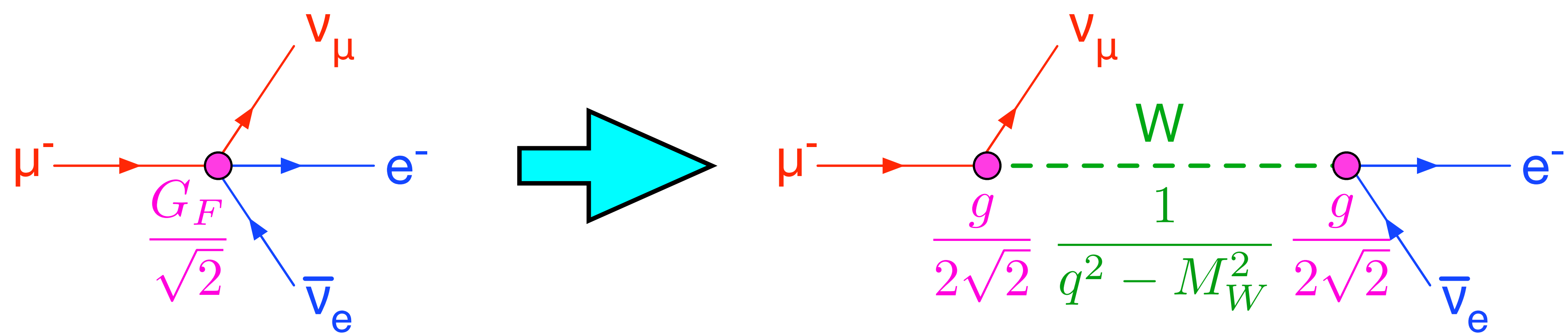
$$W^{\pm} = \frac{1}{\sqrt{2}}\left[W_1 \mp iW_2\right], Z = \frac{1}{\sqrt{g^2+(g')^2}}\left[gW_3 - g'B\right], A = \frac{1}{\sqrt{g^2+(g')^2}}\left[g'W_3 + gB\right]$$

The theory therefore has three fundamental constants: g, g', $\Phi_0 = v/(2)^{1/2}$

These are related to the other parameters as follows,

$$M_W = \frac{1}{2}gv,\ M_Z = \frac{1}{2}\sqrt{g^2+(g')^2}\ v,\ \alpha = \frac{e^2}{4\pi} = \frac{1}{4\pi}\frac{(gg')^2}{g^2+(g')^2}$$

The decay of a muon in the MSM is very similar in form to the modified version of Fermi's 1933 theory,



The 4 particle point coupling is replaced by two 3-particle couplings and a virtual W exchange. The invariant momentum transfer $q^2 < m_\mu^2 \ll M_W^2$, in the limit $q^2 \to 0$, they give identical results with the identification

$$\frac{G_F}{\sqrt{2}} = \left[\frac{g}{2\sqrt{2}}\right]^2 \frac{1}{M_W^2} = \frac{g^2}{8M_W^2} = \frac{g^2}{8(gv/2)^2} = \frac{1}{2v^2}$$

The Fermi coupling constant is completely determined by the vacuum expectation value of the Higgs field [the modern aether]!! Therefore the muon decay rate/lifetime can be expressed as a function of v

$$\Gamma_\mu = \frac{\hbar}{\tau_\mu} = \frac{G_F^2 m_\mu^5}{192\pi^3} = \frac{m_\mu^5}{384\pi^3 v^4}$$

Anyone who wants to calculate anything in the Standard Model must know g, g', and v. The most precisely measured physical quantities related to them are

Quantity	Parameter	Value	Uncertainty
(g-2)/2 of e-	$1/\alpha$	137.03599968(9)	0.7 PPB
μ Lifetime	G_F	$1.1663787(6)\times10^{-5}$ GeV^{-2}	0.5 PPM
Z mass	M_Z	91.188(2) GeV	23 PPM

The muon lifetime is a very important quantity. It provides precise information about the value of v. Our quarknet students can actually determine this fundamental parameter from their data

$$v = \left[\frac{\tau_\mu m_\mu^5}{384\pi^3\hbar}\right]^{\frac{1}{4}}$$

Today's Activity

Our job is to measure the vacuum expectation value of the Higgs field. It is responsible for producing the masses of all elementary particles. You have a stack of plastic scintillation counters, some electronics, and a laptop. Nature is providing the muons. Do your best!