AP PHYSICS LAB GUIDE

J. Patrick Polley
Department of Physics and Astronomy
Beloit College, Wisconsin

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E-mail: apintl@collegeboard.org

AP Canada
1708 Dolphin Avenue, Suite 406, Kelowna, BC, Canada V1Y 9S4
250 861-9050; 800 667-4548 (toll-free in Canada only)
E-mail: gewonus@ap.ca

Pre-AP
45 Columbus Avenue, New York, NY 10023-6992
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INTRODUCTION

The AP® Physics program has long advocated the inclusion of laboratory work in the syllabus of the Physics B and Physics C courses. One reason for this is that laboratory work can provide additional opportunities for learning, reinforcing, and exploring physical concepts that are discussed in the classroom. A second reason is that most college courses include a laboratory component in the introductory physics course. If AP Physics is to be equivalent to an introductory college course, the AP course should also include a fair amount of lab work. Students should keep a lab notebook, or other kind of lab work portfolio, that indicates the amount and the quality of the lab work that the students perform. This notebook or portfolio has been found to be useful in demonstrating to colleges the nature of the lab work undertaken in the AP course.

How much time ought to be spent on lab work? The College Board has found that lab work counts for approximately 20% of the grade in introductory physics courses, with about one lab experiment being performed each week. Many high school faculty find it difficult to set aside a double period, or two regular class periods, each week to accommodate lab work. If one lab a week is not possible, determine how much time can be spent on lab work without diluting the curriculum. Realize that, both in terms of providing a quality course and preparing the students for the exam, some lab work is necessary, but it is not necessary to have the students perform a lab exercise every week. It is better to have the students perform an exercise in depth, taking time to understand the principles involved and to analyze thoroughly the data which they obtain, than to force-march the students through a series of cookbook-style labs.

The following lab guide was written in order to provide some assistance and advice on setting up a lab component for the AP Physics B and C courses. It is not a sequence of experiments that must be followed. It does not include an array of specialized techniques that must be mastered. It does provide an introduction to data analysis and experimental procedure. There is an emphasis on graphical analysis, with some statistical analysis. The equipment used is often simple. Circuit diagrams and drawings are provided to allow teachers to build, rather than buy, equipment that is unique to some experiments. Computer acquisition and analysis of data is not emphasized, although many experiments may be carried out with computer-based interfaces and sensors. This lab guide is written less for the experienced teacher with decades of work in the classroom, and more for the neophyte teacher who needs some advice on what may be done, often with limited facilities and time, to get lab work into the AP curriculum.

After deciding how to incorporate lab work into the course, a teacher must then decide how much structure to impose on the students' lab work. At one end of the spectrum, the students manipulate lab equipment in a way that is clearly defined in a detailed procedure given to them by the teacher. The results that the students obtain are then checked against the results expected for the experiment. In this approach, the students do little if any work in setting up or designing the experiment. This approach, sometimes derisively referred to as the "cookbook" approach, is the one that is employed in the overwhelming majority of college physics courses. While it may be useful in acquainting students with the operation of equipment with which they are unfamiliar, it is largely ineffective in enhancing student understanding of the physical concepts being studied.

At the other end of the pedagogical spectrum, students engage in open investigations with minimal guidance from the instructor. The students are responsible for the design of the lab exercise, putting together the experimental apparatus, carrying out the experiment, analyzing the data, and extracting a conclusion from their analysis. For highly self-motivated and clever students, this is a wonderful way to do experimental physics. For most students, it turns into an exercise in frustration.

The middle way emphasizes student involvement in the lab by requiring them to do more than follow a tried-and-true recipe in a lockstep manner. Instructors set definite goals for the laboratory exercise, provide some of the procedure, but require students to set up laboratory equipment, or modify lab set-ups used in previous labs or classroom demonstrations. Students determine what data to take, and how to analyze it, in order to accomplish the objectives set by the instructor.

Ideally, we move the students from complete dependence on a lab procedure generated by the teacher to the point where the student can carry out independent investigations. Realistically, what can be expected in the introductory course is to begin to wean the students away from complete dependence to the point where they can perform labs along the lines of the middle way suggested above.

The labs in the AP lab guide may be presented to students in as open-ended a manner as the teacher feels comfortable with. When students first encounter some pieces of equipment, such as a voltmeter or a motion sensor, they require more guidance than they do later in the course, when they have become familiar with various apparatus. This does not mean that the kinematics labs must be done in a cookbook fashion. Many AP students have already had a course in high school physics before they take the AP course, and have some familiarity with the lab equipment. In those cases, labs early in the term can begin in a more open-ended manner.

For this reason, the AP lab guide does not mandate that experiments be done in a particular manner. The lab write-ups offered in the guide discuss the motivation for and theory behind a particular exercise, describe the procedure for carrying it out, and suggest ways of analyzing the data. **The lab exercises in this AP lab guide are NOT the canonical set of laboratory exercises that must be included in an AP Physics course.** The exercises cover many of the phenomena studied in the Physics B and Physics C courses, but the coverage is not complete. The purpose of the guide is not to present the best 19 introductory physics lab exercises. The purpose is to provide a set of exercises that illustrate the laboratory skills that are central to investigations in physics.

Safety has been a concern in determining which lab exercises to include in the guide. Most of the experiments can be undertaken knowing there is no risk to the students. Some, particularly the optics experiments that use a laser, require the teacher to adopt rigorous safety protocols. Any teacher using a laser, or any piece of equipment that uses line voltage (the 110 VAC that comes out of most wall sockets), should prepare the class for the lab with a presentation on the hazards that the lab equipment presents and the safety procedures to be used in lab. We strongly suggest that teachers take advantage of summer workshops and institutes to become familiar with equipment before using it in the classroom.

One of the questions that all physics teachers face when designing a set of laboratory exercises is the degree to which computers are used to acquire and analyze data. To some the lack of computers can be seen as a bar to doing interesting physics labs. We suggest that while a computer can be an extremely useful tool in the introductory lab, it should not become the focus of the lab activity. The emphasis should be on the physical principles being investigated, not on the manipulation of software. To that end, it is strongly recommended that students graph data by hand in the first few labs before using computer spreadsheets and graphing packages to do the work for them. The reason is that students who use only graphing programs often do not understand what such basic quantities as slope and y-intercept actually mean. The slope of a line becomes simply an answer generated by the program, and not a quantity that expresses the rate of change of one variable with respect to another. Students should be encouraged to use computers to analyze data, even if they don't use computers in the lab to acquire data, but only when their use elucidates rather than obscures the interpretation of data.

Mechanics Labs

The array of possible mechanics experiments can be overwhelming, even for experienced teachers. The mechanics experiments presented here cover one-dimensional kinematics, two-dimensional kinematics, dynamics, rotational motion, and oscillatory motion. Most of these may be done with simple equipment; one does require the use of a computer or calculator-based lab (CBL) system to acquire and analyze data.

The equipment required for most of these labs is cheap, easily transported and stored, and robust. In mechanics, the primary quantities of interest are distance, time, and mass. Each lab setup should have a meter stick or two, a stopwatch, and a set of calibrated masses. If a set of calibrated masses is beyond your budget, you might consider purchasing a few inexpensive balances.

The dynamics lab is presented in two versions, one that uses photogate timers and another that uses motion sensors. Each of the labs has some specialized equipment and materials, but an effort has been made to keep the financial outlay for new equipment small. The need to do labs in AP Physics can serve to acquire additional funds for equipment from budget-conscious administrators, but the emphasis in lab should always be on having as many students as possible participate directly in the lab exercise. It is better to have a large number of simple lab setups than a small number of gee-whiz setups.

Electricity and Magnetism Labs

Labs involving electricity and magnetism may be daunting to the new teacher, but these labs are often easy to set up and run with a minimum of equipment. The availability of cheap and reliable equipment, together with the large array of experiments that may be performed with that equipment, make E&M labs particularly attractive in the introductory course.

The following E&M labs involve two different kinds of electric and magnetic phenomena: those that are constant in time (contained in both Physics B and Physics C syllabus), and those that are time dependent (only in Physics C syllabus). The time-independent, or DC, labs that are described here require a minimum of equipment. The time-dependent labs require the use of more equipment, but allow the students to investigate more phenomena. This trade-off is one to keep in mind as you design your lab syllabus, and as you budget for equipment. Once again, as is the case for mechanics labs, it is better to have a large number of setups so that all students may directly participate in the lab, rather than a few setups that render many students passive spectators.

Each lab setup should have a solderless breadboard, a selection of hookup wires, two multimeters, and a power supply. In addition to hookup wires, the students need test leads with stackable banana plugs, as well as alligator clips that accept banana plugs. Hand tools you need: a wire stripper, needle-nose pliers, and small Phillips and straight blade screwdrivers. Your lab toolbox should also contain a roll of electrician's tape, a roll of duct tape, a soldering pencil, and a roll of thin rosin-core solder.

The hookup wire should be #22 AWG copper wire with plastic insulation. It should be available in at least two colors, and you should cut it to several lengths (2 inch, 4 inch, and 6 inch are useful) before stripping a quarter-inch of the insulation from each end of the wires. The gauge of the wire is important, as larger wire does not fit in the holes in the solderless breadboard, and smaller wires do not make good electrical contact in the holes in the breadboard.

Optics and Modern Physics Labs

The optics and modern physics labs cover material in the B curriculum. The material and equipment used is outlined in the labs, but a special note here concerning laser safety is in order. The lasers used in the introductory labs often have an output power between one and five milliwatts, which make them dangerous. A beam of that power, if it enters the eye directly, can cause severe eye injury. Students must be aware that these devices are not toys, and that misusing them can cause injury to themselves and others. The lab setups should be constricted so that there are beam stops on the lab bench (a book will do) to block the beam from bouncing around the classroom and inadvertently blinding a classmate. All lab setups should be checked to make sure that stray reflections have been minimized and contained. Safety must be emphasized when students work with these small, seemingly innocuous lasers.

The following index provides a guide to the experiments in the lab guide. The title of the experiment is given, and the objectives are listed in terms of the physical concept investigated and the lab technique taught. The labs are generally appropriate for both B and C courses, except for cases where the content is unique to one of the two curriculums.

Lab Exercise	**Concept**	**Lab Skill**
Mechanics		
Diluting Gravity 1	One-Dimensional Kinematics	Graphical Analysis Error Analysis
Diluting Gravity 2	One-Dimensional Kinematics	Computer Acquisition and Data Analysis
The Coffee Filter and Air Resistance	One-Dimensional Dynamics	Graphical Analysis
Elastic Collisions	Conservation of Momentum One- and Two-Dimensional Collisions	Error Analysis
The Turning Point	Conservation of Energy Circular Motion	Graphical Analysis Error Analysis
The Whirligig	Circular Motion	Error Analysis
Hooke's Law and Harmonic Motion	Hooke's Law	Graphical Analysis
Electricity and Magnetism (all labs can be used to teach breadboarding techniques)		
Ohm's Law 1	Ohm's Law	Graphical Analysis
Ohm's Law 2	Real vs. Ideal Test Instruments	Graphical Analysis Error Analysis
Electrical Power and Batteries 1	Internal resistance of batteries, Joule heating	Graphical analysis Numerical integration
Electrical Power and Batteries 2	Internal resistance of batteries, Joule heating	Graphical analysis Curve fitting
RC Time Constant 1	RC circuits (only in Physics C syllabus)	Graphical Analysis Error Analysis
RC Time Constant 2	RC circuits (only in Physics C syllabus)	Using an Oscilloscope Graphical Analysis
Magnetic Fields 1	Biot-Savart Law	Graphical Analysis
Magnetic Fields 2	Strength and Direction of the Earth's Field	Error Analysis Vector Algebra
Optics		
Geometrical Optics: Determining the Index of Refraction	Index of refraction of a Rectangular Slab	Error Analysis Statistical Analysis of Error Laser Safety
The Diffraction Grating	Interference	Error Analysis Laser Safety
Modern Physics		
Planck's Constant on the Cheap	Photoelectric Effect with LEDs	Graphical Analysis Error Analysis
The Photoelectric Effect	Photoelectric Effect	Graphical Analysis Error Analysis

LAB ONE

Diluting Gravity: A Computer-Based Laboratory

Introduction

The kinematics of one-dimensional motion is usually the first material covered in an introductory physics course. When confined to the case of constant acceleration, one-dimensional kinematics provides a subject that is amenable to classroom analysis and laboratory study both in non-calculus and calculus-based courses. The kinematic equations describe the position x and the velocity υ of an object in terms of its initial position x_0, its initial velocity υ_0, the time t, and the object's acceleration a. The two equations used in this lab are

$$x = x_0 + \upsilon_0 t + \frac{1}{2}at^2 \qquad (1\text{-}1)$$

$$\upsilon = \upsilon_0 + at \qquad (1\text{-}2)$$

If the object of interest is falling freely near Earth's surface, the magnitude of the acceleration is commonly denoted by g, which has a magnitude of 9.8 m/s^2. In this lab, a local value of g is measured. We dilute g by having the object accelerate down an inclined plane. We reduce the friction that the object experiences by using a glider on an air track as the object. When we raise one end of the air track so that it makes an angle θ with respect to the horizontal, the acceleration down the inclined air track is given by the equation

$$a = g\sin\theta \qquad (1\text{-}3)$$

If we start the glider from rest, Equations 1-1 and 1-2 may now be written as

$$x = x_0 + \frac{1}{2}(g\sin\theta)\ t^2 \qquad (1\text{-}4)$$

$$\upsilon = (g\sin\theta)\ t \qquad (1\text{-}5)$$

Equipment

Two ways of doing this lab are presented here. Both require an **air track** and **air track glider.** One method uses **a pair of photogates** to measure the time during which the object moves on the track. The other method uses a **sonic ranger** and **computer data acquisition system** to obtain the glider's position as a function of time. If time permits, it is useful to do the lab both ways in order to introduce students to the advantages and disadvantages of computer data acquisition.

Diluting Gravity 1

Experimental Procedure

The students find the factor $\sin\theta$ by taking the ratio of the height H to which they raise one end of the air track to the distance L between the air track supports, as shown in Figure 1.1. An easy and reproducible way to raise the track is to place a metal block under the supports at one end of the air track. A pair of blocks, one 0.5 inch high and one 1.0 inch high, allow the students to obtain data at three different track inclinations. The height of the metal blocks may be

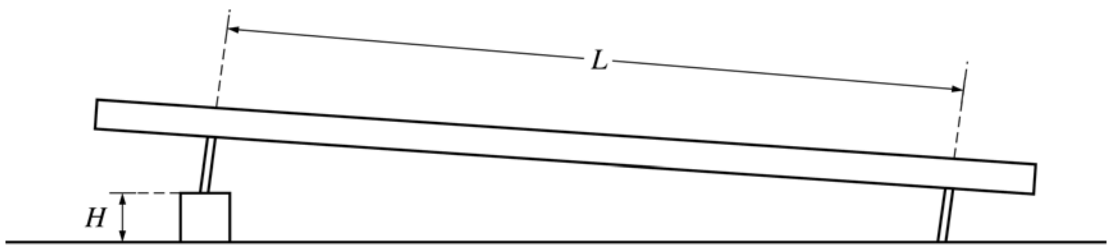


Figure 1.1

measured with a vernier caliper, and the distance between the air track supports with a meter stick. When the students calculate $\sin\theta$, they need to be careful about the number of significant figures they use. For example, suppose the distance L between the supports is 90.5 ± 0.1 cm, and the height H of the metal block is 2.531 ± 0.001 cm. The three significant figures in the measurement of L mean that the calculated value of $\sin\theta$ should have only three significant figures.

The time measurement is taken by the two photogates, which are separated by a distance S. The timer starts when the glider passes through the first gate, and stops when the glider passes through the second. The students vary the photogate separation S, and record the time it takes for the glider to traverse those separations. If the glider is released so that it triggers the first photogate as soon as the glider starts down the ramp, then Equation 1-4 may be written as

$$S = \frac{1}{2}(g\sin\theta)t^2 \qquad (1\text{-}6)$$

Thus a plot of t^2 versus S should yield a straight line with a slope equal to the reciprocal of $\frac{1}{2}g\sin\theta$. This may be used to calculate a local value of g.

Error Analysis

The two places where problems arise in this lab are the error introduced because the glider is already moving when it triggers the first photogate, and measuring the separation of the photogates. Determining the error associated with measuring the separation of the gates is straightforward. The error associated with the delayed triggering of the first photogate is a bit more involved. If, after being released, the glider travels a distance *R* before it triggers the first photogate, it has a nonzero velocity at the beginning of the interval *S*. Now we need to determine that initial non-zero velocity, to determine the size of the error it introduces. Rearranging Equations 1-1 and 1-2 to eliminate time, we obtain

$$2a(x - x_0) = v_1^2 - v_0^2 \qquad (1\text{-}7)$$

Here v_1 is the velocity as the glider reaches the first photogate, the initial velocity v_0 is zero, the acceleration *a* is equal to $g\sin\theta$, and $(x - x_0)$ is equal to *R*. The velocity v_1 is then given by

$$v_1^2 = 2gR\ \sin\theta \qquad (1\text{-}8)$$

Now assume that we want the error in the value of *S* to be less than 5% due to the nonzero velocity at the first photogate. Using Equation 1-6:

$$R = (0.05)S = (0.05)\left[\frac{1}{2}g\sin\theta\ t^2\right]$$

Setting this equal to $v_1 t$ and substituting:

$$(0.05)\left[\frac{1}{2}g\sin\theta\ t^2\right] = v_1 t \qquad (1\text{-}9)$$

$$0.025\ g\sin\theta\ t = \left[2gR\sin\theta\right]^{1/2}$$

$$R = 0.00031\ g\sin\theta\ t^2$$

For a *t* value of 4.5 seconds, and $\sin\theta$ equal to 0.1, we obtain *R* equal to 0.0063 meters, or about one quarter-inch. Students should plug in their own numbers to convince themselves that their results are affected by about this amount. They should compare this error to the effect of the uncertainty in the length measurement noted above in the discussion of significant figures.

Diluting Gravity 2

Experimental Procedure

In the second procedure the student employs a technology called the sonic ranging sensor that has been in use in physics classrooms for two decades. The sonic ranging sensor determines the distance to an object by measuring the time it takes for an ultrasonic pulse to travel from the sensor to the target and back again. Good results are obtained for flat objects that are perpendicular to the direction of propagation of the ultrasonic pulses, as long as these objects are at least 5 cm by 5 cm. This usually means that a small cardboard flag must be mounted on the air track glider to provide a flat target for the sensor beam. The sensors often have a dead spot, or region where measurements are inaccurate, which extends from the sensor out to a distance of 20 cm.

With these restrictions in mind, using the sensor is straightforward. It is best to mount the sensor at the raised end of the track, so that the glider moves further away with the passage of time. The glider is released, and then the sensor is triggered so that the computer can begin acquiring data. The data appear on the computer monitor, and may be fit to a quadratic equation using the curve-fitting program available with the data acquisition software. The coefficient of the linear term is equal to the velocity of the glider at the beginning of the data acquisition interval. The coefficient of the quadratic term is equal to $(g\sin\theta)/2$. Data are easy to acquire and analyze using this apparatus, and several trials may be run with gliders of different mass to show that the acceleration is independent of the glider’s mass.

In addition to curve fitting, data acquisition programs often allow students to differentiate the data, so that the differential of the displacement with respect to time may be displayed. Physics C students should use this feature first to obtain a velocity versus time graph, then acceleration versus time graph. When they obtain these graphs, they see that the velocity versus time graph is not a smooth linear graph with a positive slope, and the acceleration versus time graph is not a constant with respect to time. This leads to a discussion of how the computer processes the data when it differentiates numerically, and the apparent discrepancy that results when compared to the expected graphs of velocity and acceleration versus time. That discussion is useful in showing the students the power and the limitations of their data processing program. They should see that curve fitting, since it is a best fit to all of the data, is a better method of obtaining the glider’s acceleration than two successive numerical differentiations. These numerical differentiations can introduce artifacts into the data that can obscure the underlying phenomenon of motion with constant acceleration.

If students begin the lab with the working hypothesis that the glider is undergoing constant acceleration, one test of that hypothesis is to fit the data to a higher order polynomial. Students can use their data processing software to fit their data to a third-degree or higher order polynomial. The coefficients of the terms that are of cubic order or higher should be very small compared to the linear and quadratic terms. If the quadratic term is the highest significant term, this indicates that the object is undergoing constant nonzero acceleration.

LAB TWO

The Coffee Filter and Air Resistance

Introduction

Students are introduced to the concept of air resistance as a velocity-dependent force early in their study of mechanics. They are often quickly counseled to ignore it, except perhaps for the occasional problem involving the determination of a terminal velocity. If an object of mass M falls under the influence of gravity and a drag force F_{drag}, we may write Newton's Second Law as

$$Ma = Mg - F_{\text{drag}} \qquad (2\text{-}1)$$

Here a is the acceleration of the object, and Mg is the weight of the object. We may model the drag force F_{drag} as shown in Equation 2-2.

$$F_{\text{drag}} = bv^n \qquad (2\text{-}2)$$

Here the drag coefficient b is a constant that depends on the shape of the object, and v is the velocity of the object. For objects moving at low velocity through air n is usually close to 2. By substituting Equation 2-2 into Equation 2-1 we obtain:

$$Ma = Mg - bv^n \qquad (2\text{-}3)$$

As an object falls from rest the velocity increases until the drag force and the weight are equal in magnitude. The acceleration is then zero, and we have

$$Mg = b\left(v_f\right)^n \qquad (2\text{-}4)$$

This may be rearranged to give the terminal velocity v_f:

$$v_f = (Mg/b)^{1/n} \qquad (2\text{-}5)$$

Now by taking the logarithm of both sides of Equation 2-5 we obtain

$$\ln v_f = (1/n)[\ln M + \ln g - \ln b] \qquad (2\text{-}6)$$

This may be rewritten as

$$\ln v_f = (1/n)\ln M + C \qquad (2\text{-}7)$$

where C is a constant equal to $(1/n)[\ln g - \ln b]$.

We can now determine the exponent n of the velocity v if we can determine the terminal velocities v_f for a variety of masses. A log-log plot of the terminal velocity versus the mass then yields a linear plot with a slope of $1/n$.

The problem with varying the masses is that the drag coefficient depends on the size and configuration of the object. This is why we can't do the lab by dropping a ping-pong ball, a golf ball and a bowling ball from the top of the school building. That is where the coffee filters come in. By dropping stacks of coffee filters consisting of one, two, or more filters, we may vary the mass of the falling object while keeping its size and configuration approximately the same.

Experiment

The coffee filters used in this lab are those that look like an inverted crown or a basket. The corrugations of the filter make it easy to nestle them one inside the other. A large vertical drop of a few meters must be used; we suggest at least 3 meters. The best results are acquired by using as large a vertical drop as feasible.

The procedure is straightforward. A student drops the filters, while a second student times their fall. The average velocity υ of the filters is found by dividing H, the height of the drop, by T, the time of the fall from release to impact. Several trials (5-10) should be used for each value of the mass, i.e., each number of filters.

In order to find the coefficient n, we then make the assumption that the filters quickly attain their terminal velocity υ_f. We then may equate the average velocity given by H/T to the terminal velocity. As the number of filters increases, this approximation becomes worse since it takes a longer time for the larger mass to reach its terminal velocity. It is best if the students perform the experiment with groups of one, two, three, and four filters. Since the filters all have approximately the same mass, this corresponds to multiples of one, two, three, or four units of mass, with the unit of mass being one coffee filter.

The students graph the data on log-log paper, with velocity on the vertical axis and the number of filters on the horizontal axis. The slope of the resulting straight line is equal to $1/n$. The students can also graph the number of filters on the vertical axis and the fall time on horizontal axis, which will give a straight line with a slope of $(-n)$. This is a useful exercise for students who are unsure of their logarithms.

Students should be asked what effect the approximation made by equating the average and terminal velocity has on their answer for n. They should be able to reason that the average velocity is lower than the terminal velocity, and that the values of n they obtain are therefore higher that the expected value of 2.

LAB THREE

Elastic Collisions

Introduction

Collisions in one and two dimensions provide some of the most common illustrations of the conservation of linear momentum. Elastic collisions allow the student to study both conservation of linear momentum and conservation of energy. Air tracks and frictionless carts are one way of studying these collisions in one dimension. In this lab we describe how to construct a simple inexpensive apparatus for the study of one-dimensional and two-dimensional elastic collisions.

First we review the basic physics involved in elastic collisions in one dimension. We consider an object of mass M_1 moving with an initial velocity v_{1i} that undergoes an elastic collision with an object of mass M_2 moving with a velocity v_{2i}. After the impact, the mass M_1 has a final velocity v_{1f}, and the mass M_2 has a velocity v_{2f}. The general equation for conservation of linear momentum for this system states that

$$M_1 v_{1i} + M_2 v_{2i} = M_1 v_{1f} + M_2 v_{2f} \qquad (3\text{-}1)$$

where $+$ and $-$ signs are used for a specific case to indicate the direction of each velocity. When we apply the conservation of energy to this system we find that

$$\frac{1}{2}M_1 v_{1i}^2 + \frac{1}{2}M_2 v_{2i}^2 = \frac{1}{2}M_1 v_{1f}^2 + \frac{1}{2}M_2 v_{2f}^2 \qquad (3\text{-}2)$$

For the system studied in this lab, the object of mass M_2 is initially at rest. Since v_{2i} is zero, Equations 3-1 and 3-2 become, respectively, Equations 3-3 and 3-4.

$$M_1 v_{1i} = M_1 v_{1f} + M_2 v_{2f} \qquad (3\text{-}3)$$

$$\frac{1}{2}M_1 v_{1i}^2 = \frac{1}{2}M_1 v_{1f}^2 + \frac{1}{2}M_2 v_{2f}^2 \qquad (3\text{-}4)$$

We now find the final velocities v_{1f} and v_{2f} in terms of the initial velocity v_{1i} and the masses M_1 and M_2. To do this we use Equation 3-3 to write v_{1f} in terms of M_1, M_2, v_{1i}, and v_{2f}.

$$v_{1f} = v_{1i} - \frac{M_2}{M} v_{2f} \qquad (3\text{-}5)$$

Then we substitute this expression in Equation 3-4 in order to eliminate v_{1f}, and express v_{2f} in terms of M_1, M_2, and v_{1i}.

$$\frac{1}{2}M_1 v_{1i}^2 = \frac{1}{2}M_1\left(v_{1i} - \frac{M_2}{M_1} v_{2f}\right)^2 + \frac{1}{2}M_2 v_{2f}^2 \qquad (3\text{-}6)$$

After multiplying the squared term and collecting terms we obtain

$$\upsilon_{2f} = [2M_1/(M_1 + M_2)]\upsilon_{1i} \qquad (3\text{-}7)$$

If we use Equation 3-7 to eliminate υ_{2f} from Equation 3-5 we obtain

$$\upsilon_{1i} = [(M_1 - M_2)/(M_1 + M_2)]\upsilon_{1i} \qquad (3\text{-}8)$$

In this lab, for the case of objects of equal mass, Equation 3-7 implies that the second mass moves forward with a velocity equal to υ_{1i}. Equation 3-8 implies the velocity of the first mass after the collision is zero.

In two dimensions, the math is a little trickier. We must now conserve momentum in both dimensions, and therefore must write a vector equation for momentum. Assuming that υ_{2i} is zero, we have

$$\frac{1}{2}M_1\upsilon_{1i}^2 = \frac{1}{2}M_1\upsilon_{1f}^2 + \frac{1}{2}M_2\upsilon_{2f}^2 \qquad (3\text{-}9)$$

$$M_1\mathbf{v}_{1i} = M_1\mathbf{v}_{1f} + M_2\mathbf{v}_{2f} \qquad (3\text{-}10)$$

We now consider the case where M_1 and M_2 are equal. Equations 3-9 and 3-10 then become, respectively,

$$\upsilon_{1i}^2 = \upsilon_{1f}^2 + \upsilon_{2f}^2 \qquad (3\text{-}11)$$

$$\mathbf{v}_{1i} = \mathbf{v}_{1f} + \mathbf{v}_{2f} \qquad (3\text{-}12)$$

We square both sides of Equation 3-12 to obtain

$$\upsilon_{1i}^2 = \upsilon_{1f}^2 + 2(\mathbf{v}_{1f} \cdot \mathbf{v}_{2f}) + \upsilon_{2f}^2 \qquad (3\text{-}13)$$

A comparison of Equations 3-11 and 3-13 indicates that the dot product of $\mathbf{v}_{1f}$ and $\mathbf{v}_{2f}$ must be zero, which implies that the two vectors are perpendicular.

Equipment

The equipment, shown in Figure 3.1, consists of a ramp made from a piece of wooden v-molding, and a wooden support for the ramp. A holder for the target ball is attached at the end of the horizontal support as shown. The holder consists of a strip of metal that is screwed to the underside of the horizontal support, and a machine screw that is used to hold the target ball. The metal strip should not be screwed to the support very tightly, as it needs to be moved during the experiment. A nut holds the machine screw in place, and a second nut is used to hold the target ball.

The ramp support is attached to a lab bench or table with a C-clamp or table clamp so that the target support pokes out over the edge of the bench or table. The first part of the experiment

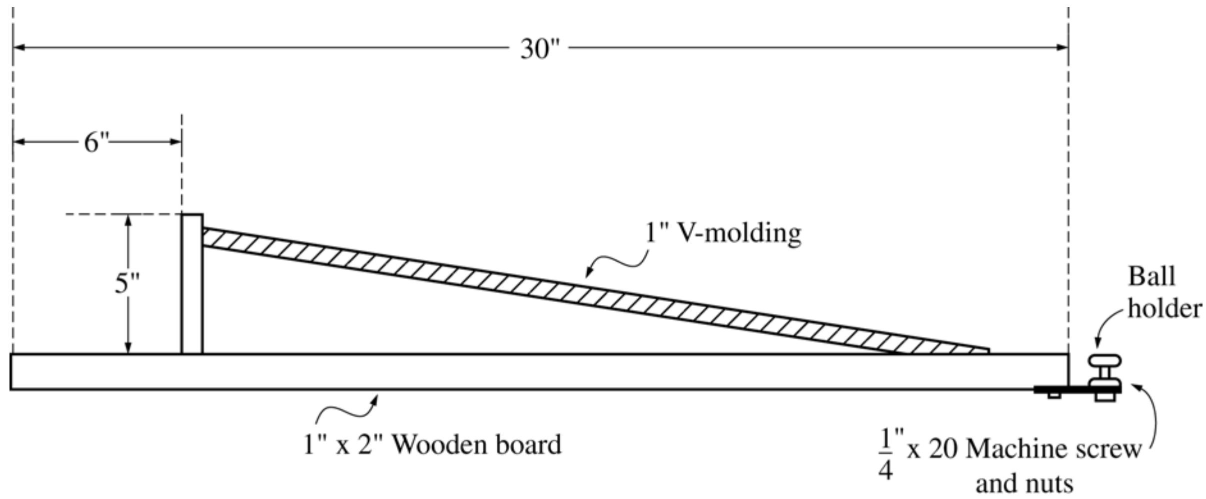


Figure 3.1

consists of rolling a ball down the ramp, with no target on the target ball holder and the holder swung out of the way. The ball leaves the support with a horizontal velocity v, and lands a horizontal distance D away. As shown in Figure 3.2, the distance D is measured from a point P directly beneath the point Q where the ball leaves the ramp, to the point of impact S. The students then cut a piece of paper large enough to extend from the point P to the point S, allowing a border of a few centimeters as well. The piece of paper should be as wide as it is long. The paper is then taped to the floor.

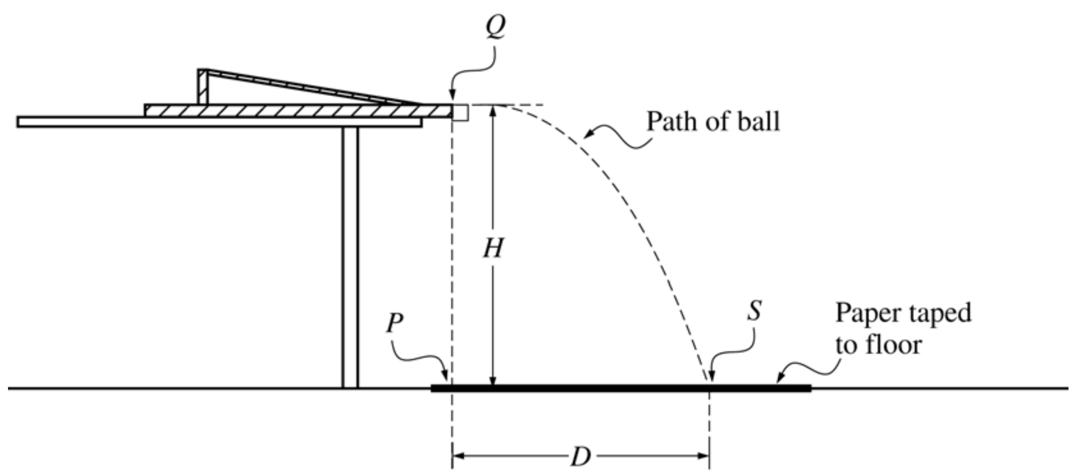


Figure 3.2

In order to record the point of impact of the ball, the students place a piece of carbon paper (yes, it is still made) face down on the large piece of paper, with another piece of paper on top of it to prevent the carbon paper from ripping when the ball hits it. The students then roll the ball down the ramp several times, letting the carbon paper mark the impact sites on the large sheet of paper.

For the collision part of the lab, it is useful to think in terms of a horizontal set of *x*-*y* axes. The *x*-axis lies along the direction of travel of the ball as it leaves the ramp. The *y*-axis is perpendicular to it. A target ball is placed on the ball holder so that the target ball will travel in the *x*-direction. Another ball of equal mass is rolled down the ramp, striking the target ball. As this collision is elastic, and the masses of the two balls are the same, the momentum of the incident ball is entirely transferred to the target ball. The target ball should land in about the same spot as the impact sites for no collision. The same method, using carbon paper, is used to mark the location of the impacts of the target ball. These marks are then compared to the first set to see if the collision was elastic.

The two-dimensional case is more interesting. The analysis of the results is made easy by the fact that when the two balls collide and move off at non-zero angles with respect to the *x*-axis, they spend the same amount of time in the air. Both balls drop through the same vertical distance *H* shown in Figure 3.2. Since both balls have no vertical velocity at the moment of impact, they fall to the ground in the same time *t*.

Figure 3.3 shows the paths of the two balls as seen from above.

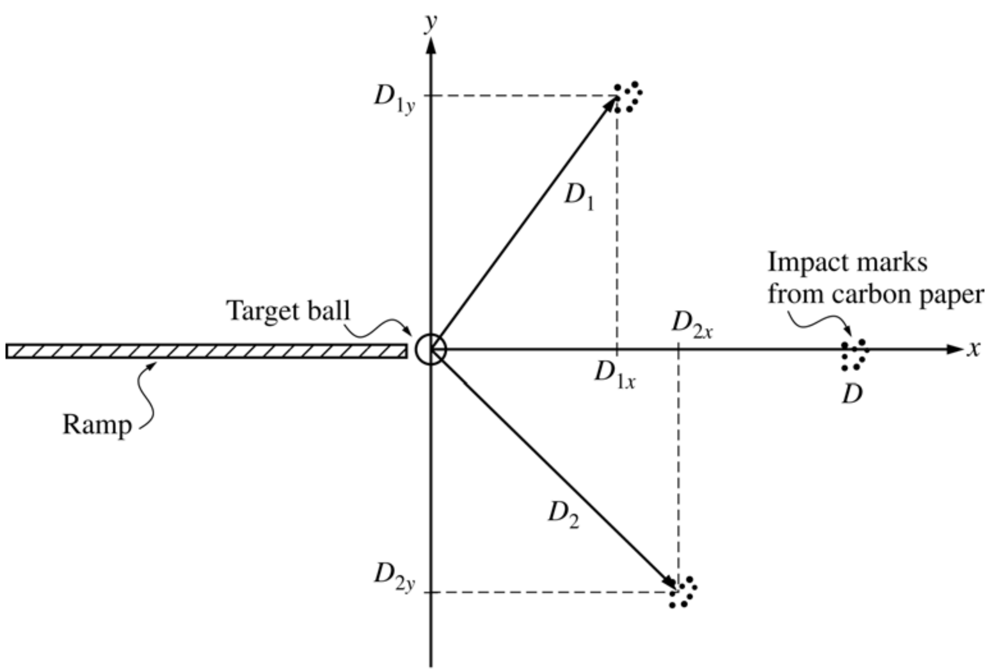


Figure 3.3

Since the horizontal components of the balls' velocities do not change until they hit the ground, the distances D_1 and D_2 are related to the horizontal velocities v_1 and v_2 by

$$D_1 = v_1 t \quad \text{(3-14a)}$$

$$D_2 = v_2 t \quad \text{(3-14b)}$$

From our earlier measurement of D, we can also determine the initial velocity υ as

$$D = \upsilon t \qquad (3\text{-}14c)$$

From conservation of momentum, we know that

$$M\upsilon = M\upsilon_{1x} + M\upsilon_{2x} \qquad (3\text{-}15a)$$

$$0 = M\upsilon_{1y} + M\upsilon_{2y} \qquad (3\text{-}15b)$$

Here υ_{1x} and υ_{2x} are the x-components of υ_1 and υ_2, and similarly υ_{1y} and υ_{2y} are the y-components of υ_1 and υ_2. We define D_{1x} and D_{2x} as the x-components of D_1 and D_2, and D_{1y} and D_{2y} as the y-components of D_1 and D_2. Substituting Equations 3-14a, 3-14b, and 3-14c into Equations 3-15a and 3-15b yields the following two equations.

$$\frac{D}{t} = \frac{D_{1x}}{t} + \frac{D_{2x}}{t} \qquad (3\text{-}16a)$$

$$0 = \frac{D_{1y}}{t} + \frac{D_{2y}}{t} \qquad (3\text{-}16b)$$

Canceling the factor of t yields

$$D = D_{1x} + D_{2x} \qquad (3\text{-}17a)$$

$$0 = D_{1y} + D_{2y} \qquad (3\text{-}17b)$$

The experiment is straightforward. The target holder is set so that the two balls go off at non-zero angles with respect to the x-axis after the collision. A ball is placed on the target holder. The other ball rolls down the ramp and collides with the target ball. The ball impacts are recorded using two pieces of carbon paper. Several trials are then run with the target holder in the same position. After a suitable number of collisions are run, the target holder is then repositioned, and another set of collisions are run at the new target holder position. Two sets of data are sufficient, but more can be collected.

The students then determine the centers of the sets of impacts that they obtain, and see if the distances D_{1x}, D_{2x}, D_{1y}, D_{2y}, and D agree with the predictions of Equations 3-17a and 3-17b. The students should attempt to determine what the errors associated with their data are, and incorporate those errors into their determination of the agreement of their data with the predicted results. The scatter of impact marks on the sheet results in an uncertainty associated with the x and y measurements. When adding the x- and y- components of D_1 and D_2, the students should be sure to carry these uncertainties along in their calculations.

LAB FOUR

The Turning Point

Introduction

The Turning Point is based on a problem commonly encountered in homework sets in the Physics B and Physics C courses. The situation is pictured in Figure 4.1. A pendulum is held out so that the pendulum's string is parallel to the ground. The pendulum bob is released, and it describes a circular arc of length L as it descends. When the bob passes through its lowest point, the pendulum string encounters a peg P located a distance D below the pivot point O of the pendulum. The bob continues its motion, but in an arc of reduced radius R. The bob follows a circular arc, passing above the peg P at a point S. The problem, as usually formulated, is to find the minimum distance D for which the string will remain taut as the bob passes through S.

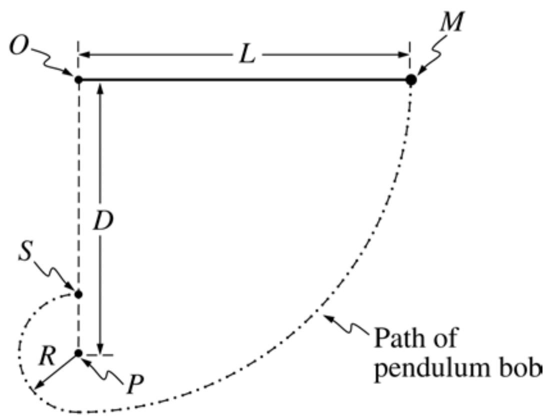


Figure 4.1

Applying Newton's Second Law, the condition on the string as the bob passes through S may be represented by Equation 4-1.

$$Ma = T + Mg \tag{4-1}$$

Here M is the mass of the bob, T the tension in the string, a the acceleration of the bob, and g the acceleration due to gravity. Since the bob is undergoing circular motion as it passes through S, we have Equation 4-2.

$$a = v^2/R \tag{4-2}$$

Here v is the velocity at point S, and R is the radius through which the bob is moving. At the minimum distance D, the tension T goes to zero, so we may substitute Equation 4-2 into Equation 4-1 to obtain

$$Mv^2/R = Mg$$

This simplifies to Equation 4-3.

$$v^2 = Rg \tag{4-3}$$

Therefore we can find R, and thus D, if we know v. Upon examining Figure 4.1 we see that we may apply conservation of energy at point S to determine v. If we take the zero of gravitational potential energy as the lowest point of the bob's path, we may write Equation 4-4

$$MgL = Mg(2R) + Mv^2/2 \tag{4-4}$$

Substituting Equation 4-3 into Equation 4-4 yields

$$MgL = 2RMg + MRg/2$$

This simplifies to

$$R = 2L/5$$

Examination of Figure 4.1 also shows that the distance D is the difference between L and R, so we have Equation 4-5.

$$D = 3L/5 \qquad (4\text{-}5)$$

Procedure

The equipment for the lab consists of a **ring stand** or **long rod** about a meter in length, **two right-angle clamps**, **two short rods**, **string**, and a **mass** such as a **rubber stopper** or **lead weight**. If you use a long rod as the vertical support for the apparatus, you should fasten it to the table with a **table clamp** for stability.

The lab set-up is straightforward. Fix the vertical rod to the table with a clamp or set the ring stand up on the table. At the top of the vertical rod, attach one short rod using a right-angle clamp. Attach the other short rod to the vertical rod about halfway up the vertical rod. This second rod is the pivot rod that the pendulum string strikes as the bob moves through its arc. The apparatus is shown in Figure 4.2.

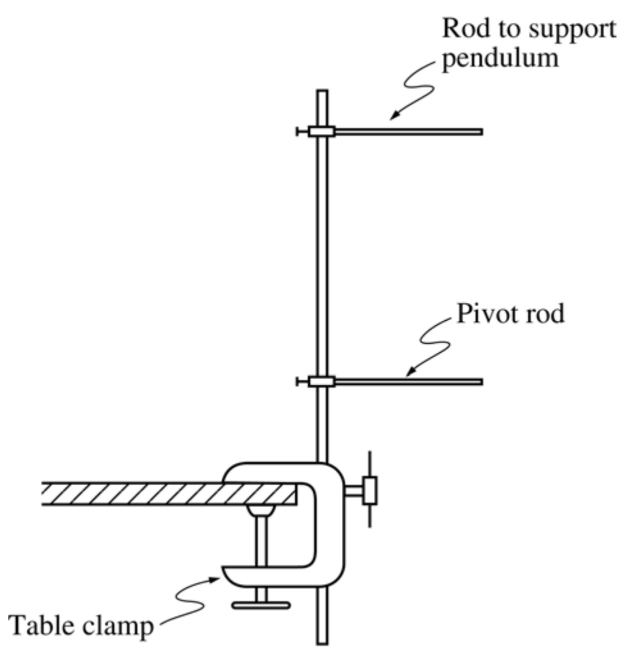


Figure 4.2

Care must be taken in attaching the string to the support rod. The string should not stretch much and should be easily tied (monofilament fishing line works well). It is best to drill a hole close to the end of the rod through which the string may be passed, rather than attempting to tie the string around the support rod. This prevents the movement of the point of attachment of the string while the bob is in motion. It also makes it easier to change the length of the string, so that data may be taken at different values of L. (Special pendulum clamps designed for lab experiments are also available commercially.)

There are several ways to do the experiment. One is to have the students calculate the highest position at which the lower rod must be placed so that the bob rotates about the lower rod while keeping the supporting string taut. They place the lower rod at the distance, release the bob from a horizontal position as indicated in Figure 4.1, and see if the bob describes the desired path. If the bob does not, the students adjust the position of the lower rod to obtain the desired path. Even if the bob does follow the desired path, the students should move the lower rod to convince themselves that they have found the *minimum* distance D for which that path occurs.

A more systematic approach is to start with the lower rod at a distance of 0.75 L below the upper rod, then decrease D in small increments until the bob revolves around the lower rod with the string taut. The amount by which D is decreased should be a few millimeters for each trial. One question that the students should answer before they start their measurements and trials is how to measure the distance D. Should they measure to the middle of the lower rod, to its bottom, or to its top? Which measured distance more accurately represents the distance used in the derivation?

If there is enough time, students should repeat the experiment for a range of values of L, preferably from 0.4 meters to 0.9 meters, to generate a table of L and D values. If D is plotted versus L, the data should fall on a straight line with a slope of 0.6. This procedure eliminates some systematic errors that may be present in students' procedure. For instance, if L or D is measured in such a way that the measurements are always too short or too long compared to their actual values, the graph will eliminate those systematic errors, as it depends on the rate of change of D with L. Students should compare the value of the slope of the D versus L plot with the values they obtain for the ratio of L and D for each trial. If the slope is close to the expected 0.6 value, while the ratios are not, this indicates that the students have a systematic error in their data. The students should address this discrepancy in their write-up of the experiment.

LAB FIVE

The Whirligig

Introduction

In this lab the students investigate circular motion in the horizontal plane using an object attached to a string, as shown in Figure 5.1. The forces acting on the object are also shown in Figure 5.1. If the motion is in the horizontal plane, then the vertical component of the tension T and the force of gravity must cancel. In the horizontal plane, the horizontal component of the tension must equal the centripetal force acting on the object. This may be written as Equation 5-1.

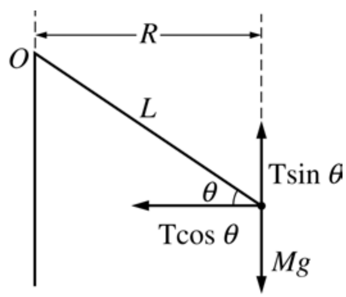


Figure 5.1

$$T\cos\theta = Mv^2/R \qquad (5\text{-}1)$$

Here v is the speed of the object, and R is the radius of the circle through which it moves. If the string that attaches the object to the pivot point O is of length L, then R may be written as Equation 5-2.

$$R = L\cos\theta \qquad (5\text{-}2)$$

If the object revolves with a period t, then the object's velocity may be written as Equation 5-3.

$$v = 2\pi R/t \qquad (5\text{-}3)$$

Substituting Equation 5-3 into Equation 5-1 yields

$$T\cos\theta = 4\pi^2 MR/t^2 \qquad (5\text{-}4)$$

Substituting Equation 5-2 into Equation 5-4 yields

$$T\cos\theta = 4\pi^2 ML\cos\theta/t^2 \qquad (5\text{-}5)$$

This simplifies to Equation 5-6,

$$T = 4\pi^2 ML/t^2 \qquad (5\text{-}6)$$

In this lab the students measure T with a force sensor or spring scale, and compare it to the value of T calculated using Equation 5-4.

Procedure

The experiment requires a specialized piece of apparatus that may be constructed or purchased. The apparatus consists of a long thin metal tube, about a meter in length. The mass consists of a large, somewhat soft object, such as a rubber stopper. Use of a deep-sea fishing sinker, or other hard object, is strongly discouraged for safety reasons. The string or monofilament line is attached to the object and then run through the tube. A force sensor, spring scale, or weight hanger to which various numbers of weights are added, is then attached to the string at the bottom of the tube as shown in Figure 5.2.

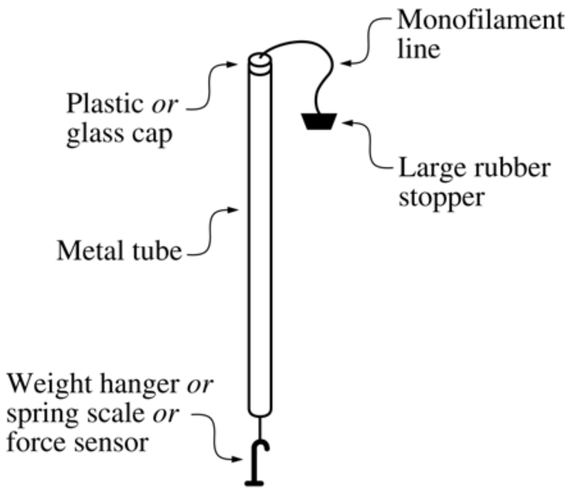


Figure 5.2

If you decide to make this apparatus, it is a good idea to put a small Teflon or glass cap at the top end of the tube, in order to reduce friction between the string and the tube. It is also a good idea to remove any sharp edges on the glass or plastic caps; otherwise the whirling mass will make an unscheduled, unexpected, and often unpleasant transition from circular motion to two-dimensional projectile motion.

This experiment can easily take the full attention of three students. This is a good thing, as it means fewer pieces of the apparatus are operating in the lab room than if the students worked in two-student teams. One student should operate the whirligig. The second student should monitor the force sensor, spring scale, or weight hanger. The third student times the rotation of the object. The most difficult part of the lab is whirling the object in such a manner that it undergoes uniform circular motion.

The procedure for performing the experiment is straightforward. The mass M of the object is measured. A length L is selected, and the apparatus adjusted to obtain that value. The whirligig operator sets the object in motion, maintaining as constant a value of the tension T as possible. Once the student monitoring the force is satisfied with the value and stability of the tension, the time may be determined. The third student, using a stopwatch, measures the elapsed time for a set number of revolutions of the object around the pole. For convenience, this set number is often 10 or 20. The period t is then found by dividing the entire elapsed time for those revolutions by the number of revolutions.

As an example, let's take a 0.150 kg object attached to the string, where L is 0.80 meters. A student whirls it so that the tension reading is 4.1 newtons. The timer records an elapsed time of 10.98 seconds for 10 revolutions. The calculated tension is given by

$$\begin{aligned} T_{\text{calc}} &= 4\pi^2(0.150\text{ kg})(0.80\text{ m})\big/(1.098\text{ s})^2 \\ &= 3.93\text{ newtons} \end{aligned}$$

Clearly 4.1 newtons, the measured value of T, disagrees with 3.93 newtons, the calculated value of T. At this point the question of significant figures and experimental error arises.

The disagreement E between the calculated tension T_{calc} and the measured tension T_{meas} is given by

$$E = (T_{\text{meas}} - T_{\text{calc}})\big/T_{\text{meas}}$$

In this case, we find E to be

$$E = (4.1 \text{ s} - 3.9 \text{ s})/4.1 \text{ s}$$
$$= 0.04 \text{ or } 4\%$$

If we examine the sources of error in the measured quantities, we may associate errors with the measurement of length, mass, and time. The length measurement should be accurate to within 1 or 2 millimeters. The relative error E_{length} associated with the length is then

$$E_{\text{length}} = \Delta L/L$$
$$= 0.002 \text{ m}/0.800 \text{ m}$$
$$= 0.0025 \text{ or } 0.25\%$$

The mass measurement should be accurate to within a gram. The relative error E_{mass} associated with the mass is then

$$E_{\text{mass}} = \Delta M/M$$
$$= 0.001 \text{ kg}/0.150 \text{ kg}$$
$$= 0.0067 \text{ or } 0.67\%$$

The error associated with the time is the trickiest to determine. If the absolute error associated with the time was plus or minus one unit of the smallest measured division of time measured with the stopwatch, the absolute error in this case would be 0.01 seconds for the 10 revolutions. A better estimate may be obtained by having all of the lab groups simultaneously measure the time it takes for 10 revolutions of a single apparatus. The groups then write down their times, and take one-half of the difference between the lowest and highest times as the absolute error associated with the time measurement. Suppose that in this lab, with four lab groups, the following times were recorded.

Group Number	Time (seconds)	t (seconds)
1	10.98	1.098
2	10.74	1.074
3	11.42	1.142
4	11.02	1.102

The absolute error Δt is then given by

$$\Delta t = (1.142 \text{ s} - 1.074 \text{ s})/2$$
$$= 0.034 \text{ s}$$

The relative error E_{time} is given by

$$E_{\text{time}} = 2\,\Delta t/t$$
$$= 2(0.034 \text{ s})/1.098 \text{ s}$$
$$= 0.06 \text{ or } 6\%$$

The factor of two occurs since the time appears as a squared quantity in the equation for the tension. If we add the errors together to obtain the total error E_{total}:

$$\begin{aligned} E_{\text{total}} &= E_{\text{mass}} + E_{\text{length}} + E_{\text{time}} \\ &= 0.25\% + 0.67\% + 6\% \\ &= 7\% \end{aligned}$$

If we treat these independent errors more properly, and add them quadratically, we have

$$\begin{aligned} E_{\text{total}} &= \left[E_{\text{mass}}{}^{2} + E_{\text{length}} + E_{\text{time}} \right]^{1/2} \\ &= 6\% \end{aligned}$$

In either case, the calculated value for the tension and the measured value for the tension do agree to within the experimental error.

Data may also be acquired by making several measurements of the period of revolution at different values of the tension. The student then plots the measured tensions T versus the reciprocal of the square of the time in order to obtain a linear plot (see Equation 5-4). The slope of the line should be equal to the quantity $4\pi^2 ML$. Either method provides the students with experience in error analysis, and can be chosen on the basis of the time available for the lab.

LAB SIX

Hooke's Law and Harmonic Motion

Introduction

Robert Hooke, nemesis of Isaac Newton, noted that the force necessary to extend an elastic object is proportional to the distance the object is stretched. The force exerted by the object when stretched is opposite to its extension, giving us the vector equation

$$\mathbf{F} = -k\mathbf{x} \tag{6-1}$$

where k is the elastic constant, which for a spring is typically called the spring constant. Now suppose that an object of M is attached to a spring of spring constant k. Further suppose that the spring is extended or compressed and then released. From Newton's Second Law we have the following equation of motion for the object.

$$M\left(d^2x/dt^2\right) = -kx \tag{6-2}$$

This may be rearranged to yield Equation 6-3.

$$\left(d^2x/dt^2\right) = -(k/M)x \tag{6-3}$$

Equation 6-3 is of the form

$$\left(d^2x/dt^2\right) = -\omega^2 x \tag{6-4}$$

where ω is the angular frequency of oscillation, and is related to the frequency of oscillation f by Equation 6-5.

$$\omega = 2\pi f \tag{6-5}$$

Equation 6-4 describes any kind of simple harmonic motion. The solutions to Equation 6-4 are of the form given in Equation 6-6

$$x(t) = A\sin(\omega t + \phi) \tag{6-6}$$

where A is the amplitude of the oscillation, t is the time, and ϕ is the phase angle.

Using the fact that the period of oscillation T is the reciprocal of the frequency, we may use Equations 6-3, 6-4, and 6-5 to obtain the following relation between the period of oscillation, the spring constant k, and the oscillating mass M.

$$T = 2\pi(M/K)^{1/2} \tag{6-7}$$

This is the relation that the student verifies in this lab.

In order to do the lab, the student needs to know the value of the spring constant. A straightforward way of doing this consists of adding a weight to the end of the spring and seeing how far it stretches. If the spring stretches a distance L when a mass M is attached to it then from Equation 6-1 we have

$$Mg = kL \tag{6-8}$$

The spring is stretched with a variety of masses, and the extensions *L* measured for each value of the stretching masses *M*. A graph of *L* versus *M* then yields a straight line with a slope equal to g/k. The spring constant may then be calculated from the slope.

Equipment

Each lab setup should consist of a **meter stick**, a set of **calibrated masses**, a **spring**, a **stopwatch**, and a **support** for the spring. The support can be as simple as a dowel or rod that hangs over the edge of a table. Be sure to tape or otherwise fix the support in place before doing the lab.

Procedure

The lab proceeds in three steps. First, the students determine the spring constant *k* by hanging various masses from the spring at rest and measuring the extension of the spring. Second, the students determine the spring constant *k* by hanging a variety of masses from the spring and setting them in motion. By plotting the period *T* against the square root of the masses, the spring constant *k* may be determined from the slope of the resulting straight line. In the third part, the two values of *k* are compared, and any discrepancies resolved.

In the first part of the lab, the unstretched length of the spring is recorded. A mass is added to the spring, and the new spring length is recorded. The extension of the spring is the absolute value of the difference between these readings. The students continue by adding masses to the spring, and determining the spring's extension. The students must always calculate the extension of the spring with reference to the unstretched length.

As the students measure the lengths and masses, they should also make note of the experimental error introduced into their measurements. If the marks on the meter stick are one millimeter apart, then the error involved in each measurement is uncertain by half a millimeter, and the error involved in the extension is twice that. Thus an extension of 25 millimeters should be reported as 0.025 ± 0.001 meters. Wherever possible, students should render all measurements in SI units. When the students graph the extension versus the masses, they should indicate this uncertainty in the form of an error bar on the data point.

The error involved in the mass measurements is more difficult. If the students use a balance, then the error may be assumed to be equal to plus or minus 1 of the last significant digit measured by the balance. Thus if the balance read 54 grams, the mass would be reported as 0.054 ± 0.001 kilograms.

As the students collect data for their graphs, they will ask how many data points they must take. Rather than giving them a number, ask them to think about how many values they think they will need for their graphs, and what values they will need. Ask them if it is better to take measurements at 1-gram increments from 10 to 20 grams, or fewer measurements, e.g., 20-gram increments from 20 grams to 100 grams. Which set of measurements is more useful, and why? This will get the students thinking about interpolation and extrapolation of data, and begin to get them beyond simply "cookbooking" the lab.

Once the students have their data, they should plot the data by hand to obtain their extension versus mass graphs. The students should then obtain the slope of the linear plot they have, and extract the value of the spring constant from the slope using Equation 6-8. One mistake that students commonly make is to use a single point on the graph, rather than actually calculating the slope of the best-fit straight line via dividing its rise by its run.

Students then proceed to the second part of the lab, where they determine the spring constant *k* by setting a mass in oscillation and timing its period of oscillation. Students should attach a mass to the spring, then extend the spring a short distance vertically. The amplitude of motion should be small enough that the spring does not become slack as it oscillates. A common mistake in timing the oscillations is to count a full cycle of oscillation every time the mass passes through its equilibrium point. Students do better if they count full cycles of oscillation when the mass reaches one end point of its trajectory, at the maximum extension or compression of the spring. Students should time several full cycles rather than attempting to time just one cycle. They should be able to explain why this procedure reduces the error involved in determining the period of oscillation.

Students should determine the period of oscillation for several values of mass, for example, 0.050 kg, 0.070 kg, 0.100 kg, 0.0150 kg, and 0.200 kg. The masses should be different enough so that a difference in the period is readily apparent. Students should then graph the period versus the square root of the mass in order to obtain a linear plot. The slope of this line may then be determined, and the spring constant obtained using Equation 6-7.

There should be a discrepancy between the two spring constant values. This results from not taking into account the mass of the spring itself. The correction for the mass values used in the second part of the lab consists of adding one third of the spring's mass to the attached masses. The effective mass, M_{eff}, is given by

$$M_{\text{eff}} = M + \left(M_{\text{spring}}/3\right) \tag{6-9}$$

The derivation of this correction term is often omitted in textbooks, and since it uses calculus it is only appropriate for the Physics C course. The kinetic energy of the spring may be written as the integral over the infinitesimal mass elements *dm* from zero to the spring's full length *L*:

$$K_{\text{spring}} = (1/2)\int (dx/dt)^2\, dm \tag{6-10}$$

Now the ratio of the velocity υ of the mass attached to the spring to the velocity (dx/dt) of the mass element dm located at x is given by

$$\upsilon/L = (dx/dt)/x \tag{6-11}$$

The mass density ρ is given by

$$\rho = dm/dx \tag{6-12}$$

or, in terms of the mass of the spring M_{spring} and the length L

$$\rho = M_{\text{spring}}/L \tag{6-13}$$

Substituting Equations 6-11, 6-12, and 6-13 into Equation 6-10 yields

$$K_{\text{spring}} = \frac{1}{2}\int \frac{M_{\text{spring}}\upsilon^2 x^2}{L^3}\,dx \tag{6-14}$$

Integrating x from zero to L yields

$$K_{\text{spring}} = \frac{1}{2}\;\frac{M_{\text{spring}}\upsilon^2}{L^3}\;\frac{L^3}{3} \tag{6-15}$$

This may be written as

$$K_{\text{spring}} = \frac{1}{2}\;\frac{M_{\text{spring}}}{3}\;\upsilon^2 \tag{6-16}$$

Thus the kinetic energy of the spring plus the mass M may be written as

$$K = \frac{1}{2}\left(M + \frac{M_{\text{spring}}}{3}\right)\upsilon^2 \tag{6-17}$$

An extension of this lab consists of doing a log-log plot of the data in order to determine the exponent in Equation 6-7. Students may take the same number of points, or more to extend the range of the data, and plot the logarithm of the period versus the logarithm of the mass. Taking the log of both sides of Equation 6-7 yields

$$\log T = \frac{1}{2}\log M - \frac{1}{2}\log k + \log(2\pi) \tag{6-18}$$

Thus a plot of $\log T$ versus $\log M$ should give a straight line with a slope of $1/2$. Students may do this plot using a spreadsheet program, or using a sheet of log-log graph paper. If the students have not done a log-log graph before, it is better to have them do this by hand and calculate the slope from the graph. Too many students who rely entirely on software to do all of their graphical analysis do not understand how to calculate a slope from a log-log graph.

The students then have a value of the spring constant k from the method described in the first part of the lab, and they know that the period is proportional to the square root of the mass. What they don't know is that the period is proportional to the reciprocal of the square root of the spring constant. Ask the students if they can think of a way to show this experimentally.

Another extension of the lab involves demonstrating that the period of oscillation is independent of the amplitude of oscillation. One way of verifying this is to have students perform the determination of the period of oscillation at a variety of amplitudes. A better check is to assign each lab pair or team a different amplitude to check, then have them perform that measurement at each of the lab setups. After all lab pairs or teams have rotated through all of the setups, the results can be tabulated and discussed. This should lead to a discussion of systematic errors in measurements, for some of the lab groups will be consistently lower or higher than everyone else as a result of flaws in their procedures.

LAB SEVEN

Ohm's Law

Introduction

The laboratory exercises in this section study Ohm's Law, which states that the voltage drop V across an electrical component of resistance R that has a current I through it is given by $V = IR$. In this lab the student varies the current I through a resistor R, and measures the resulting changes in the voltage drop V across the resistor. The advantages to this exercise is that data are easy to acquire, and once acquired, a graph of the data result in a linear plot that is easy to interpret.

Ohm's Law 1

Procedure

The lab setup is straightforward. The power supply may be as fancy as a variable-voltage variable-current supply, or as simple as a D cell in a battery holder. The internal resistance, or output impedance, of the power supply is assumed to be negligible in this lab, compared to the other resistances in the circuit.

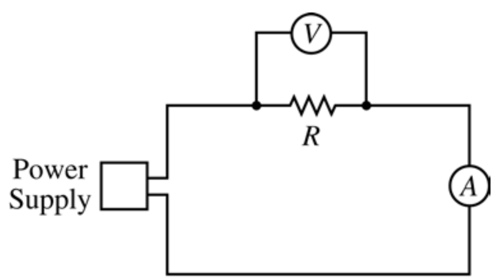


Figure 7.1

If the power supply has a variable output, the circuit is as simple as that shown in Figure 7.1. Varying the current through the resistor could be as simple as turning a knob on the front panel of the power supply. However, we suggest that the variable output feature of the power supply not be used in this lab, in order that the operation of the circuit is as transparent as possible to the students.

The current can be varied by setting up the circuit shown in Figure 7.2. There are two resistors in the circuit, R_1 and R_2, where R_1 is the resistor under study. By changing the value of R_2, the current that flows through R_1 is varied.

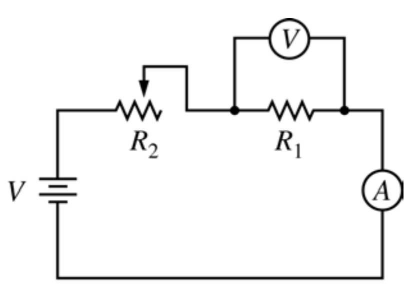


Figure 7.2

There are two ways to vary R_2. One is to use a variable resistor, or potentiometer, often called a pot. If you choose this option, solder one wire to the wiper, or middle terminal of the pot. Solder another wire to one of the two remaining terminals. If you set a multimeter to the ohmmeter scale and attach the multimeter to the pot before the pot is inserted in the circuit, the students can then see that the resistance of the pot does vary as the shaft is turned.

The resistance of the pot should be an order of magnitude larger than the resistor that you plan to use for R_1. For example, if R_1 is a 1 kilohm (1 k) resistor, then the pot should have a value of 10 kilohms (10 k). This value refers to the maximum value of the pot's resistance, and is usually marked on the rear of the pot's case.

The second method of varying R_2 is to use a set of resistors. For a 1 k value of R_1, a set of six R_2 values that give you fairly evenly spaced values of current and voltage are 0.0 (a wire), 470 ohms, 1 k, 2.2 k, 4.7 k, and 10 k. It can be useful to have the students work out the expected voltage and current values before they actually do the lab.

Before starting, students should make a data table in which they enter the values of current and voltage that are obtained. Students should wire the circuit up themselves. It is often useful, particularly when students work with the solderless breadboards for the first time, for them to have a working example in the lab room against which to check their work. As the students wire up their circuits, ask them to not make a complete circuit with the battery or to turn on the multimeters until you check their work. Once you are assured that their connections are correct, have them complete their circuits and turn on the multimeters.

The multimeters should always be set on the lowest sensitivity, or highest full-scale voltage setting, before they are turned on. The meter sensitivity may then be increased, one scale at a time, until the meter reads the maximum number of significant figures. While most multimeters have an automatic overload indicator, this practice of starting the experiment with the equipment at low sensitivity is an important one to teach, as it can prevent the destruction of sensitive equipment in other contexts.

Once the students acquire the data, they should graph it on linear graph paper and determine the resistance of R_1 from the slope of the best-fit straight line. The value that they obtain is then compared to the value of R_1 as measured by a multimeter in the ohmmeter mode. To measure R_1, break the circuit by disconnecting one of the power supply leads, and switch the multimeter that was being used as a voltmeter to its ohmmeter mode.

Ohm's Law 2

Procedure

This experiment is an extension of Ohm's Law 1, in which the internal resistances, or input impedances, of the voltmeter and ammeter are explored. The idealizations of an ammeter as having zero internal resistance and of a voltmeter as having an infinite internal resistance are often useful. These assumptions simplify the description of the behavior of circuits into which these two pieces of test equipment are inserted. The careful experimentalist remembers that these are assumptions, and is aware of the restrictions that the equipment used in an experiment imposes.

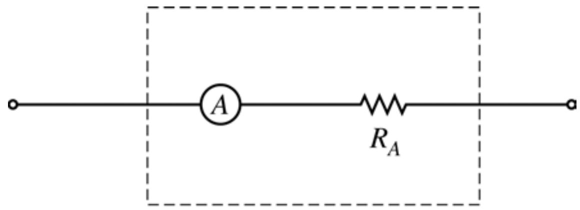


Figure 7.3

A real ammeter can be represented as an ideal ammeter in series with a small resistor R_A as shown in Figure 7.3. The ideal ammeter measures the current flowing through it while offering no resistance to that current. The resistor R_A represents the small but finite resistance with which any real ammeter opposes the flow of that current. This resistance is on the order of a fraction of an ohm, so that it may be ignored in many applications.

In order to measure R_A, we need to be able to measure resistances on the order of an ohm. One way to do this is shown in Figure 7.4. Resistor R_1 is a 100 Ω resistor that limits the current through the circuit. Resistor R_2 represents one of a set of resistors whose values run from 0.47 Ω to 10.0 Ω. In this first exercise, students should use the voltmeter to measure the voltage drop across points *AB* and across points *AC*. A suggested approach is to calculate the value of the ammeter resistance R_A for five values of R_2, such as 0.47 Ω, 1.0 Ω, 2.2 Ω, 4.7 Ω, and 10 Ω. Students should comment on the resulting values for R_A that they calculate, and discuss which they believe to be most accurate and why.

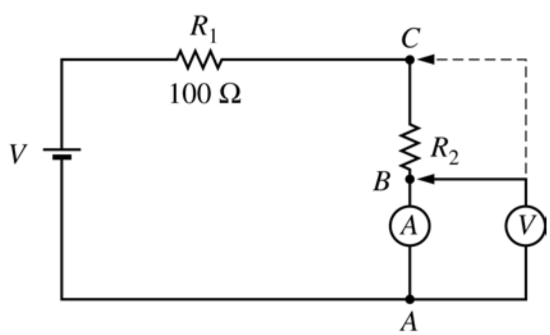


Figure 7.4

The second part of the exercise involves studying the internal resistance of the voltmeter. Most digital multimeters have an internal resistance of a couple of megohms. If you are using old VOMs (**v**olt-**o**hm-**m**eters) in your lab, please be aware that their internal resistance is on the order of tens of kilohms.

For this lab a 9-volt battery is a good choice for a power source, because you use little current in the lab. The circuit shown in Figure 7.5 allows the student to investigate the internal resistance R_V of the voltmeter, where R_1 is a resistor that varies

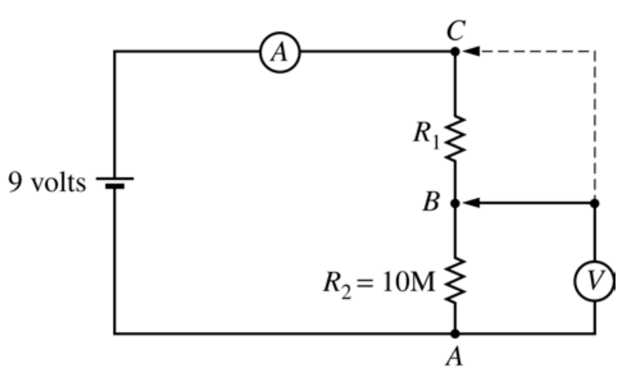


Figure 7.5

from 2.0 to 20 megohms. Among the values of R_1 that are the easiest to assemble are 2.0 MΩ, 3.3 MΩ, 5.0 MΩ, 10 MΩ, 15 MΩ, and 20 MΩ. We suggest that students be asked to construct these resistors from 10 MΩ resistors, using the appropriate combinations in series and parallel. The student measures the voltage across *AB* and *AC*, and the current through the ammeter for those two voltage measurements (should the current be the same for the two measurements?), and uses the measured values of current and voltage to determine R_V, the internal resistance of the voltmeter. Once again, the student should be asked which value of R_V is most accurate, and why.

LAB EIGHT

Electrical Power and Batteries

Experiment 1

Electrical power is an easy concept for students to master, particularly as it applies to DC circuits. Students learn early on that the power P dissipated in a circuit element is given by

$$P = IV \tag{8-1}$$

Here I is the current through the circuit element, and V is the voltage drop across it. When Equation 8-1 is combined with Ohm's Law, either the current or voltage can be eliminated from the equation to obtain

$$P = I^2 R \tag{8-2}$$

$$P = V^2/R \tag{8-3}$$

where R is the resistance of the circuit element.

The battery, though ubiquitous, is more difficult to understand. All students know that batteries run down, but see little explanation of this common and annoying fact in their physics courses. Most students are familiar with the concept that the battery has an internal resistance. This resistance limits the output current available from the battery, and can model the drop in the battery's output voltage with increasing current.

What this model doesn't do is explain why the battery runs down. In this lab the student models the behavior of the battery, without attempting to develop an explanation based on the chemistry of the battery. Modeling is often a useful first step in understanding physical processes. Modeling the battery involves coming up with a reasonably accurate equation that can predict the behavior of the battery as a function of load and time.

In this lab the student drains a small battery (AAA) with a large load so that the battery runs down appreciably in the course of an hour. The circuit is straightforward, and is shown in Figure 8.1. The load resistor R is a small resistor in the range from 4.7 to 10 Ω. It is a good idea to have a variety of power ratings available for each value of R that the students use, so that they must calculate the resistor power rating that is adequate for the experiment. The students should be able to use Equations 8-1 to 8-3 listed above to see why using a $1/4$ watt, 4.7 Ω resistor with a 1.5 volt battery is not a good idea.

The battery is placed in a small **AAA battery holder.** The **voltmeter** in Figure 8.1 measures the voltage drop across the **ammeter** and the **load resistor *R***. The student should be able to explain why the current through the voltmeter may be ignored in the calculation of the power output of the battery. The circuit should not be closed until the instructor checks all the connections, and the students are ready to proceed. The circuit may be kept open either by leaving one of the leads from the battery holder unconnected, leaving the battery out of the holder, or inserting a small switch in the circuit between the voltmeter and the battery.

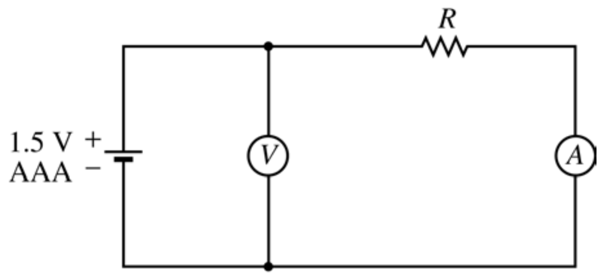


Figure 8.1

Once the students are ready to take data, they complete the circuit and read the voltmeter and ammeter simultaneously at fixed time intervals. Taking the data at intervals of one or two minutes for 45 minutes to an hour generates enough data for the lab. Data acquisition can be done by computer, but this is a lab where there is enough for three members of a lab group to do. The meters will not record steady or steadily decreasing values. The values displayed by the meters are generally decreasing, but there are fluctuations in the voltage and current readings. As the students take the data, they should try to watch the fluctuations in the current and the voltage, and use a method agreed on by the lab group to figure out what reading they will use.

After the data is acquired, the students need to calculate the power from their readings of voltage and current. The students should display their data in tabular form, being careful to use appropriate units throughout. The calculated values of the power are then plotted versus time, either by using a spreadsheet program such as Excel, or by hand. The lab groups then determine what the best-fit curve to their data looks like.

After the data have been graphed, the students then determine how much energy from the battery was dissipated in the course of the lab. The energy output is given by the integral of the power over time, which is the area under the curve that they have plotted. The easiest way to calculate this area is using a trapezoidal approximation, which is illustrated in Figure 8.2. For those not familiar with this numerical method, a short description is given here. We begin with values of the power P calculated at equal time intervals t. In the lab there are N such time intervals. There are $N + 1$ values of the power, starting from the initial value P_0 and ending with the final value P_N. The energy E_{i+1} dissipated in the interval t_i is given by the area under the curve between points P_i and P_{i+1}. This value is given by the trapezoidal rule as

$$E_{i+1} = [(P_i + P_{i+1})/2]t_i \qquad (8\text{-}4)$$

In this experiment the time intervals t_i all have the same values.

Students may wonder about the accuracy of this procedure. Figure 8.2 might indicate to them that there is an inherent error involved, and they would be right. The error Δf involved in numerically integrating a function $f(x)$ over the interval $[a, b]$ is given by

$$\Delta f = \left(h^2/12\right)(b-a)\left(d^2 f(\zeta)\big/dx^2\right) \qquad a < \zeta < b \qquad (8\text{-}5)$$

(i.e. the second derivation of *f* is evaluated at ζ). Here *h* is the width of the sections into which the interval $[a, b]$ is subdivided.[1] Students should note that the size of the error is directly proportional to the square of the width of the time interval. Reducing the time interval by a factor of two can reduce the error by a factor of four. There are more sophisticated methods available to calculate the area under the curve, such as Simpson's rule, but the trapezoidal method is accurate enough for this lab. The reason is that the data themselves have an error associated with them that swamps the error ΔE.

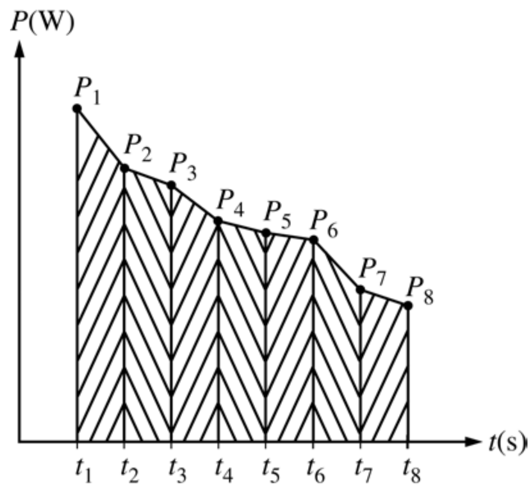


Figure 8.2

Experiment 2

In the second lab the students fit a curve to data generated in the same manner as in Experiment 1. The second experiment may be done with the data collected in Experiment 1, or with a new set of data collected in a second lab period. This lab points up the difference between two approaches to curve fitting that we often take in lab.

Usually when we fit a curve in lab, we rely on a good "eyeball" fit. We expect roughly the same number of points to be above and below our best-fit curve. We also attempt to minimize the distance between the points and the curve that we fit to the data. Sometimes, when we know or expect the data to fit a straight line, we calculate the least-squares fit for a linear relationship between the independent and dependent variables in the data. Such calculations are easily done with a spreadsheet program, or with a programmed calculator.

Problems arise when the data do not fall neatly on a linear plot, a log-log plot, or a semilog plot. When this is the case, students can still usually draw a curve freehand that seems to fit the data well. This is often adequate for purposes of interpolating data. But there is no analytic

[1] B. Carnahan, H. A. Luther, J. O. Wilkes, *Applied Numerical Methods*, (John Wiley and Sons, 1969, 70-72.

function that would allow them to extrapolate from their data. Of course, any extrapolation is beset by problems. Questions arise such as the applicability of the experimental conditions to values of the experimental variables outside the measured range. Nonetheless, extrapolation and prediction are often what scientists are called upon to do. This exercise provides a useful introduction to the problems extrapolation involves.

The students set up Experiment 2 just like Experiment 1, taking data at five-minute intervals for 45 to 50 minutes. This gives the students 10 or 11 data points. Unless something has gone wrong in the experiment, the data will show the output power of the battery decreasing in time in a nonlinear manner. The students need to answer the following two questions.

(a) What is the analytical form of the best fit to this data?

(b) Using their analytical fit, what is the predicted power output of the battery in 24 hours?

If students have access to a spreadsheet program that will provide log-log and semilog plots, they can obtain analytical fits in the form of Equation 8-6 for semilog plots.

$$P = A\exp(-t/\tau) \tag{8-6}$$

Alternatively for log-log plots they obtain Equation 8-7.

$$P = Bt^n \tag{8-7}$$

In Equations 8-6 and 8-7, *A* and *B* are constants, τ is the time constant for the exponential decay, and in this case, *n* is a negative number. Once the data has been plotted and fit, the students should use their fit to predict the battery output power 24 hours later. They leave the circuit connected, and the next day check the actual power output of the battery with their predicted value.

If a spreadsheet program with curve-fitting capabilities is not available, students can perform the lab using semilog paper for the exponential decay, and log-log paper for the power law decay. They do this by making the best linear fit to the data in these two cases. The slope of the line on the semilog paper gives the reciprocal of the time constant τ. The slope of the line on log-log paper gives the exponent *n* in Equation 8-7.

This still leaves the problem of how to determine the constants *A* and *B*, which requires some calculus. One method is to integrate the area under the curve analytically and fit it to the value given by the numerical integration. The first step in this process is to integrate the area under the power versus time curve from 5 minutes to 50 minutes using the trapezoidal rule outlined in Experiment 1. The second step is to perform the same integration analytically, using the values of τ and *n* from the semilog and log-log graphs respectively. The constant *A* may be found by setting the analytic result for the exponential function, which involves the factor *A*, equal to the numerical result. The constant *B* may be found by setting the analytic result for the power law integration, which involves a factor of *B*, equal to the numerical result.

Now that the students have determined the values of the constants for Equations 8-6 and 8-7, they may make their predictions. As noted above, one prediction may be made for the power output after 24 continuous hours of operation. The students might also make predictions for the power output at the 2-hour mark, and compare that to the predicted value, if time allows. All of the data analysis need not be done before the data are taken at either the 2- or 24-hour intervals. The agreement of the measured power output at the 24-hour mark will probably not agree very well with the predicted value, while the 2-hour agreement is usually better.

LAB NINE

RC Time Constant
(Only in Physics C Syllabus)

Introduction

Circuits containing a resistor and a capacitor connected in series are among the easiest circuits exhibiting time-dependent behavior for students to investigate. A simple circuit for investigating this time-dependent behavior is shown in Figure 9.1. When the switch *S* is closed, the circuit is complete and current begins to flow, charging the capacitor *C*. The voltage drops around the loop must equal the charging voltage V_0. We write this as

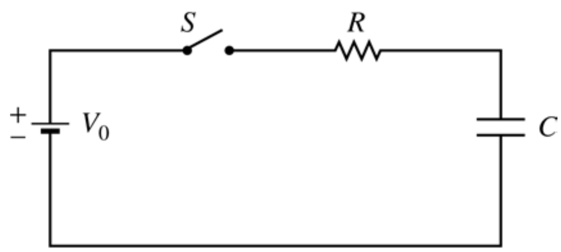


Figure 9.1

$$V_0 = V_C + V_R \qquad (9\text{-}1)$$

Here V_C is the voltage drop across the capacitor *C*, and V_R is the voltage drop across the resistor *R*. We set V_C equal to Q/C, where *Q* is the charge on the capacitor. We also set V_R equal to *RI*, using Ohm's Law. Substituting these two expressions into Equation 9-1 we obtain

$$V_0 = Q/C + RI \qquad (9\text{-}2)$$

Now we exploit the fact that the current *I* is the time derivative of the charge flowing through the resistor *R* to obtain the first-order differential equation

$$V_0 = Q/C + R(dQ/dt) \qquad (9\text{-}3)$$

Solving this differential equation for *Q(t)* we obtain

$$Q(t) = V_0 C\left(1 - e^{-t/RC}\right) \qquad (9\text{-}4)$$

Differentiating this expression with respect to time gives the current *I* as a function of time:

$$I(t) = (V_0/R)e^{-t/RC} \qquad (9\text{-}5)$$

The voltage drop V_R across the resistor *R* is given by $RI(t)$, so we have

$$V_R(t) = V_0 e^{-t/RC} \qquad (9\text{-}6)$$

We see that the voltage across the resistor is exponentially decreasing. In this lab we measure the voltage $V_R(t)$ to see if it corresponds to the voltage predicted by this equation.

Experimental Procedure #1

This experiment is carried out using the circuit shown in Figure 9.2. The **voltmeter** is a **digital multimeter (DMM)** with an internal resistance of several megohms. The **battery** is a **1.5 volt D cell**. The resistor is a $\frac{1}{4}$ W resistor with a resistance between 10 kilohms and 100 kilohms. The capacitor is a **large electrolytic capacitor** with a capacitance of 1000 μF to 10,000 μF. The students should be given the capacitor and asked to determine the resistance required to make the product of resistance and capacitance approximately 100. From Equation 9-5 it can be seen that *RC* has units of time, and is equal to the time required for the voltage to drop to $1/e$ of its original value. The product *RC* is referred to as the *time constant* for the circuit. In this case, a value of 100 seconds for the time constant gives a reasonable charging and discharging time for the experiment.

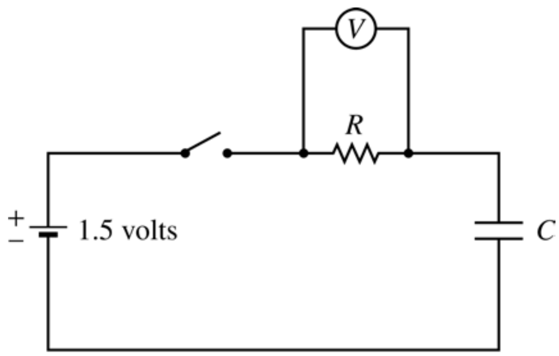


Figure 9.2

The reason for giving the students the capacitor first and asking them to find the value of the resistor is that big capacitors are large and expensive items while resistors are cheap. It is more cost effective to start with a single size of capacitor, bought in quantity for this lab, than to buy an array of large capacitors that find little additional employment in the introductory physics course.

To carry out the experiment, students close the switch *S* in Figure 9.2, and read the voltmeter every 10 seconds until the voltage goes to zero. The data may then be plotted as voltage versus time to show the exponential decrease of the voltage drop across the resistor with time. The data are then plotted a second time on a semi-natural log plot of voltage versus time. The slope of this second graph is equal to $(-1/RC)$.

Students may then compute the time constant *RC* from the two graphs and compare the two values they obtain. To obtain the time constant from the first graph, students may draw a best fit curve to the data, then locate the point on the curve where the voltage is equal to $1/e$ of its initial value. The time when that occurs is the time constant *RC*. The time constant is found on the second graph by determining the slope, then setting that value equal to $(-1/RC)$.

After the two values of *RC* are determined, students may then determine the actual value of the capacitors used in the lab. The digital multimeters may be used as ohmmeters to measure the actual value of the resistors, and then the capacitance may be determined from the time constant *RC*. Students may then compare the nominal value, that is the value printed on the capacitor, to the value they calculate. They should determine if the value they obtain lies within the stated tolerance of the nominal value. For example, suppose the students use a 10 kilohm resistor, and obtain a time constant of 116 seconds. This yields a capacitance of 11,600 μF. If the capacitor they are using has a nominal value of 10,000 μF ± 20%, this means the time constant may be expected to yield a value between 8000 μF and 12,000 μF. Thus the value of the capacitance calculated from the experimental data agrees with the nominal value of the capacitor, within the stated tolerance.

Experimental Procedure #2

This experiment requires an **oscilloscope**, a **function generator**, a **solderless breadboard**, a **resistor**, and a **capacitor**. This equipment is wired up as shown in Figure 9.3. The oscilloscope is used to measure the input waveform to the *RC* circuit and the waveform across the resistor. The function generator supplies the input waveform, in this case a square wave. The capacitor charges and discharges once during each complete cycle of the square wave. This charging and discharging causes a current to flow through the resistor. The voltage drop across the resistor due to the current flowing through it is displayed on the oscilloscope.

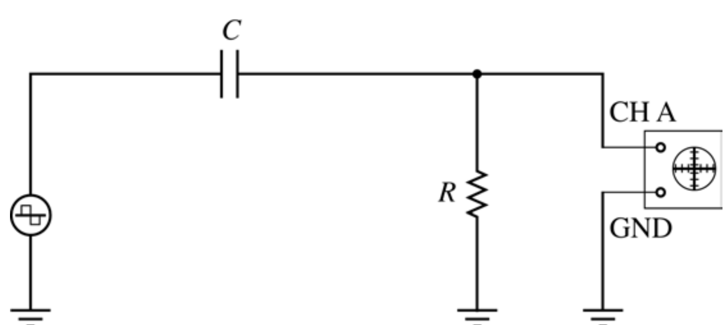


Figure 9.3

Now we consider some actual numbers, in order to anticipate what we will see on the oscilloscope. It is a good idea to have the students do this as well, so that they think through the behavior of the circuit. We begin by assuming that the function generator is set to 1 kilohertz. We further assume that the value of the resistor is 1.5 kilohms, and the value of the capacitor is 0.1 μF. The *RC* circuit has a time constant equal to the product of the values of the resistor and capacitor. For the values given above, the time constant is 0.15 milliseconds.

The function generator supplies a voltage $+V$ during the charging phase of the 1 kHz signal. If the square wave is symmetrical, that is, if the voltage is equal to $+V$ during one-half of the period of the wave and zero during the other half, the charging voltage is supplied for

0.5 milliseconds. This allows the capacitor to charge up to 97% of $+V$. When the input voltage drops to zero during the second half of the 1 kHz square wave, the capacitor discharges. The students can observe the charging and discharging cycle easily on the oscilloscope. The grid on the face of the oscilloscope is used to obtain the time that the capacitor takes to charge to $1/e$ of the input voltage. This time is equal to the time constant for the RC circuit, and can be compared with a value for the rise time calculated from the nominal values of the resistor and the capacitor.

The resistors used in this lab commonly have a tolerance of $\pm 5\%$. The tolerances of the capacitors are often great, ranging from 10% to 50%. It is a good idea to determine the tolerances of the components and to inform the students what those tolerances are, before they compare their experimental values to the value expected from the nominal values of the resistor and capacitor.

Equipment Tip

Since some classes may not have access to function generators, a simple circuit is provided in Figure 9.4 that supplies a square wave with a frequency of 1 kHz. The circuit uses an integrated circuit relaxation oscillator, two resistors, a capacitor, and a 9-volt battery.

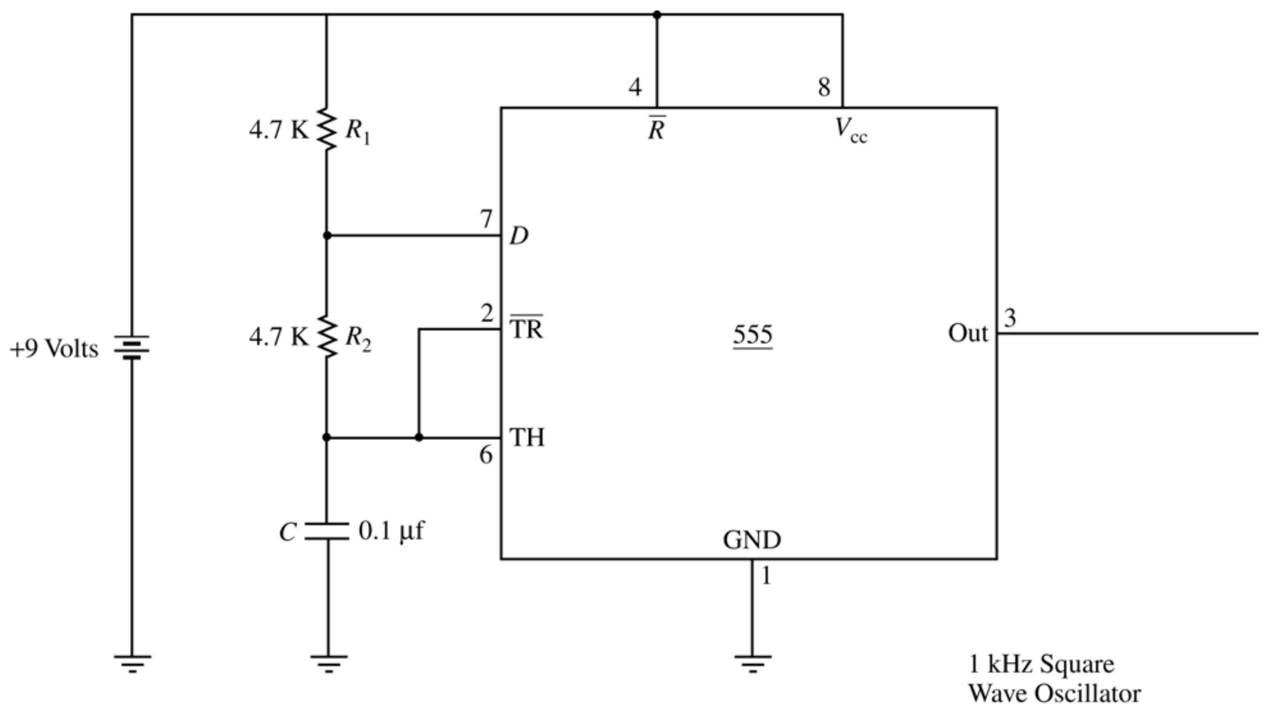
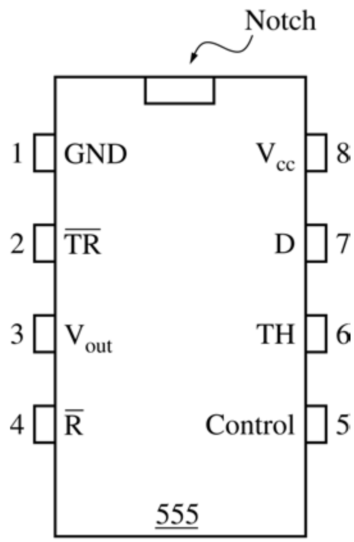


Figure 9.4



555 Pin-Out

Figure 9.5

The oscillator chip, a 555, is inexpensive and readily available. The battery is the most expensive component in the circuit, as the total cost of the other components is less than a couple of dollars. A greatly enlarged top view of the oscillator chip, and an identification of its pins, is supplied in Figure 9.5. The period *T* of the oscillator is given by Equation 9-7.

$$T = 0.693(R_1 + 2R_2)C \qquad (9\text{-}7)$$

The oscillator may be constructed on the same solderless breadboard that you use to construct the *RC* circuit.

LAB TEN

Magnetic Fields*

Introduction

Labs in which students made more than qualitative investigations of the properties of the magnetic field have been more difficult to put together until the recent advent of inexpensive Hall-effect probes. In this lab two lab exercises involving the measurement of magnetic fields are described. The first involves the measurement of the magnetic field due to a current loop. The measurement can be undertaken either with a commercially available unit, or with an inexpensive gaussmeter whose construction is described here. The gaussmeter has a resolution of ± 0.1 μT. This gaussmeter is used in the second lab exercise to study the direction and magnitude of the Earth's magnetic field.

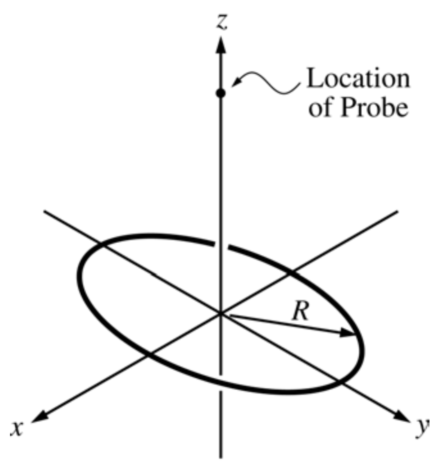


Figure 10.1

Theory

The problem of determining the magnetic field due to a current loop is a standard one. We posit a loop of radius R carrying a current I and lying in the x-y plane, as in Figure 10.1, and determine the field along the z-axis. After applying the Biot-Savart law, we find that the field $B(z)$ is given by Equation 10-1.

$$B(z) = (\mu_0 I/2)\left[R^2 \middle/ \left(R^2 + z^2\right)^{3/2}\right] \qquad (10\text{-}1)$$

If we have a coil of N turns, Equation 10-1 becomes

$$B(z) = N(\mu_0 I/2)\left[R^2 \middle/ \left(R^2 + z^2\right)^{3/2}\right] \qquad (10\text{-}2)$$

The difficulty of carrying out a lab involving this situation may be seen by examining some typical numbers. Let us suppose that the radius is 0.1 meter, that the current is 1 amp, and that the coil consists of 100 turns. The field at the origin is only 6.2 gauss, or 0.62 mT. At a z value of 0.1 meters, it is 2.2 gauss, or 0.22 mT. These fields are difficult to detect without a specialized piece of apparatus.

One way of getting around this problem of small fields is to use an AC current in the coil, and to monitor the field using a small pickup coil. The idea is that the current in the big coil

* This lab requires the teacher or students to construct some electronic apparatus. Teachers with little or no experience with electronics may wish to forego this lab, or to undertake it as an extended project with students.

induces a current in the small coil, which may be monitored on an oscilloscope. The two difficulties are that the pair of coils has a coupling that is frequency dependent, and that the phase between the signals in the primary and secondary are also frequency dependent. There is also the question of whether it is wise, pedagogically, to treat an AC circuit as though it was a DC circuit.

Magnetic Fields 1

Procedure

The equipment needed for this lab is a **coil**, a **power supply**, a **meter stick**, a **ring stand**, an **ammeter**, and a **magnetic field probe**. The coils may be made beforehand, or the students may make them as part of the lab exercise. Large diameter ($R \geq 0.1$ meter) coils are good, as they reduce any errors that come from having the probe slightly off the *z*-axis. If the power supply may supply 1 to 2 amps, a coil of 100 to 200 turns gives a good strong B field that will be much larger than the Earth's field out to a distance of $2R$ to $3R$.

The coil may be wound on any circular form of the desired radius. Cake pans are good examples. After the students wind the desired number of turns, they should carefully remove the coil from the form, and then wrap the coil with tape to prevent it from immediately transforming itself into a gigantic tangle of wire.

The coil may be driven directly from the power supply if the power supply is current limited. Otherwise, a load resistor must be used to limit the current drawn by the coil. The load resistor must have a power rating adequate for the current being drawn by the coil. Calculating the size of the power resistor needed for this task should be one of steps that students complete to prepare for the lab.

If you don't have a power supply that can provide an adequate current, a 6-volt lantern battery may be used. Again, the calculation of the size of the needed load resistor is an essential part of the lab.

The coil is now laid flat on the table. The magnetic field probe is attached to the ring stand with a clamp, and lowered until the probe is in the center of the coil. It may be necessary to raise the coil above the ring stand base in order to get the probe into the center of the coil. This can be done by laying the coil on a large book. The coil is energized, and the field strength measured. The current is then changed, either by adjusting the power supply or by changing the resistance of the load resistor, and the field is measured again. This procedure is repeated until the students have 5 or 6 values of the magnetic field at different current values. This part of the lab demonstrates the linear dependence of the field on the current in the coil.

The current in the coil is then set to its maximum value. The probe is raised 0.02 meters, and the magnetic field measured again. This procedure is repeated until the probe is about 3 R above the plane of the coil. This data is used to verify Equation 10-2.

The coil circuit is then opened, so that no current flows in the coil. The magnetic field is again measured at every point from $z = 0$ out to $z = 3R$ where data was taken initially. These background readings are subtracted from the data taken on the first run in order to obtain the field due to the coil alone. It is best to measure the background field at all points, because there may be significant magnetic anomalies in the lab arising from ferrous ring stands, magnetized tools, or permanent magnets that are in the drawers of the desk below the surface of the lab table or bench.

Analysis

The first part of the analysis is graphing the current versus field data. This data should yield a straight line with a slope of $N\mu_0/2R$. The students should be asked if it makes a difference in their determination of the slope whether the background value is subtracted or not.

The second part of the graphical analysis is more demanding. The students need to first subtract out the background values from field readings. They then plot the values of the coil's magnetic field versus the distance *z*. They may rewrite Equation 10-2 as

$$B(z) = D\left(R^2 + z^2\right)^{-3/2} \qquad (10\text{-}3)$$

Here the constant *D* is given by

$$D = N\mu_0 IR^2/2 \qquad (10\text{-}4)$$

The value of the field measured at $z = 0$ is used to fix *D*. Using Equation 10-3, a curve is then drawn of the theoretical values of the magnetic field. The experimental data are plotted to see if they fall on this theoretical curve. The agreement between theory and experiment is very good.

Yet another graphical exercise is to plot the magnetic field versus $\left(R^2 + z^2\right)^{-3/2}$. This plot generates a linear fit whose slope is given by *D* in Equation 10-4. Since the number of turns *N*, the current *I*, and the radius of the coil *R* are all known, the graphical *D* value may be compared with the *D* value computed from these known values.

In making this comparison, the students should consider sources of error in *D*. The number of turns is known to ± 1 turn, and the current is known to within $\pm 2\%$ with most ammeters. The radius *R* is the most uncertain of the values, and the students should think carefully about how to determine *R* and its uncertainty. The most straightforward method is to average the inner and outer diameter of the coil across four or five diameters. The error that results from these measurements should be taken into account when the two values are compared. The total error from *N*, *I*, and *R* is often about 5%, and the two *D* values usually agree to within that accuracy.

Apparatus

The apparatus is designed to allow students to measure magnetic fields with a sensitivity of $\pm 0.1\ \mu\text{T}$. It requires a minimum of electronic construction skills, and takes about 2 hours to build. The circuit may be built on a solderless breadboard as part of the student lab, or constructed and housed in a small cabinet or box as a permanent piece of apparatus.

The gaussmeter is built around an inexpensive Hall-effect device, the 3515 Linear Hall-Effect Sensor◆. The sensor requires a +5 volt supply, and provides a signal that is centered at around 2.5 volts. The output of the 3515 is 50 mV/mT with an accuracy of ±10%.

The sensor is mounted on a small piece of circuit perfboard. A piece $\frac{3}{8}$ inch wide and 1.5 inches long works well. The leads for the signal, +5 volt supply, and ground should be 24 AWG copper wire or smaller. Thicker wires make it difficult to maneuver the probe easily. The probe may be glued or epoxied to the perfboard for a more stable mounting. The probe is designed so that a magnetic field coming into the device from below results in an increase in the signal voltage, while a magnetic field coming into the device from above results in a decrease in the signal voltage. Here "above" refers to the side of the 3515 with lettering on it.

If you want a probe to use in determining the magnetic field in a long solenoid, the easiest way to do it is to mount the 3515 on the end of a wooden dowel so that the lettering on the 3515 faces outward from the end of the dowel. The dowel may then be inserted into the solenoid, and the magnetic field measured as described above.

◆ See "Measurement and Analysis of the Field of Disk Magnets," Martin Connors, *The Physics Teacher*, May 2002, pp. 308-311.

The circuit is shown in Figure 10.2. The output from the 3515 goes to an op amp U_1 that is configured as an inverting amplifier with a gain of 10. The output from U_1 goes to U_2, a

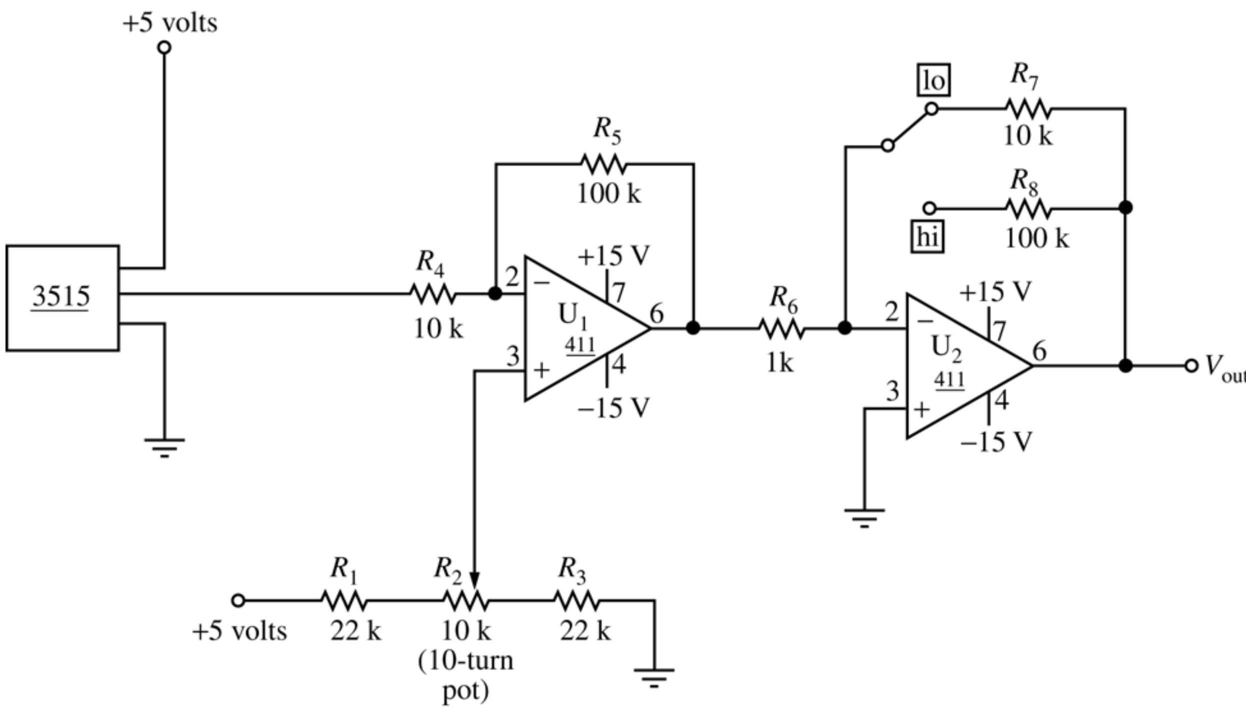


Figure 10.2

second op amp configured as an inverting amplifier with a gain of either 10 (low setting) or 100 (high setting), depending on which resistor is in the feedback loop. This increases the sensitivity of the probe by a factor of 1000 when used at the high setting, or 50 mV/μT.

The variable voltage divider consisting of R_1, R_2, and R_3 requires some explanation. The output from the 3515 consists of excursions around a voltage of 2.5 volts. If this output from the probe were simply amplified, it would quickly pin the op amp output to the supply voltages of positive or negative 15 volts. The variable voltage divider allows the first op amp U_1 to subtract off that 2.5 volt baseline voltage, and amplify only the departures from that baseline.

The probe output is temperature dependent, as it is based on a solid-state Hall-effect sensor. The op amps U_1 and U_2 may be any general-purpose op amp, but 411s are better than 741s. The op amps should be mounted in sockets, as soldering directly to the op amp leads may easily damage the op amps.

The output of the circuit goes to a digital multimeter used as a voltmeter. As the multimeter has a sensitivity of at least ± 5.0 mV, the probe has a sensitivity of ± 0.1 μT.

The power supply voltages are ± 15 volts, and +5 volts. Some lab power supplies or electronics trainers have these outputs readily available. If not, the supply shown in Figure 10.3

is easy to construct. It consists of three regulator chips, a 7815, a 7915, and a 7805, that provide +15 volts, − 15 volts, and +5 volts respectively, from a set of four 9-volt batteries. It is best to remove the batteries from the unit when it is not in use. The advantage to using batteries is that the unit is now portable.

The operation of the probe unit is simple. It is best to start with the gain selector set to low. The unit is turned on, and the zero-adjust potentiometer R_2 is adjusted to null the reading on the multimeter. The gain may then be switched to high, and the zero-adjust pot tweaked to null the output.

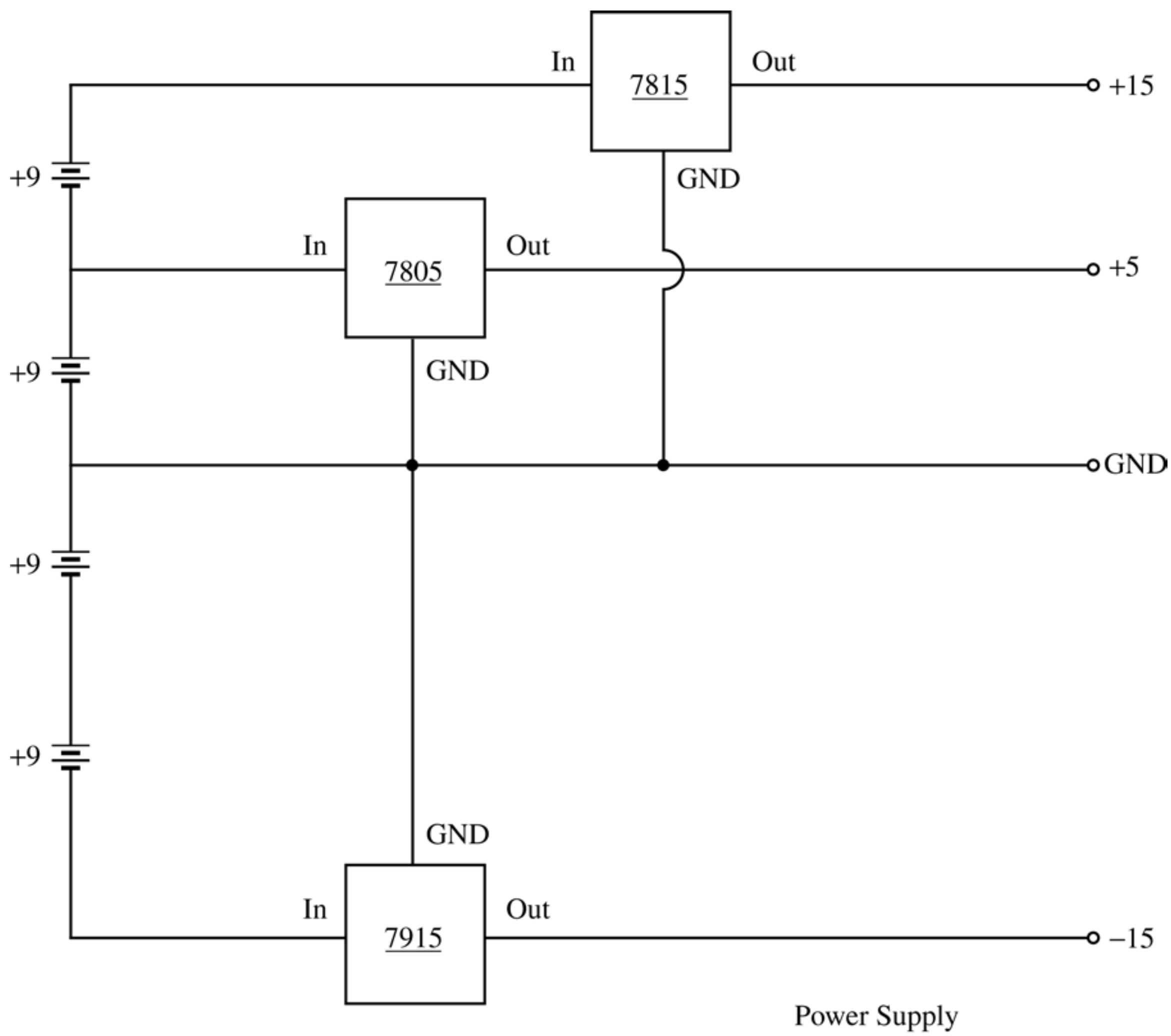


Figure 10.3

Magnetic Fields 2

Procedure

This method for obtaining the magnitude of the Earth's magnetic field and the dip angle is straightforward.

For this lab exercise the students need a **gaussmeter** with a sensitivity of 0.1 μT, such as the one described in Magnetic Fields 1. The probe should be one that is easy to level. A **spirit level** is adequate to determine the orientation of the probe. A **non-magnetic ring stand and clamp** are also needed.

The gaussmeter probe described above should be mounted on a wooden dowel that is about 6 inches long. The dowel is then mounted in a ring stand clamp and set about 24 inches above the lab bench or table. The ring stand is taped to a rectangular piece of cardboard. A ruler is taped to the lab bench, and the cardboard on which the ring stand rests is positioned snugly next to it, as shown in Figure 10.4.

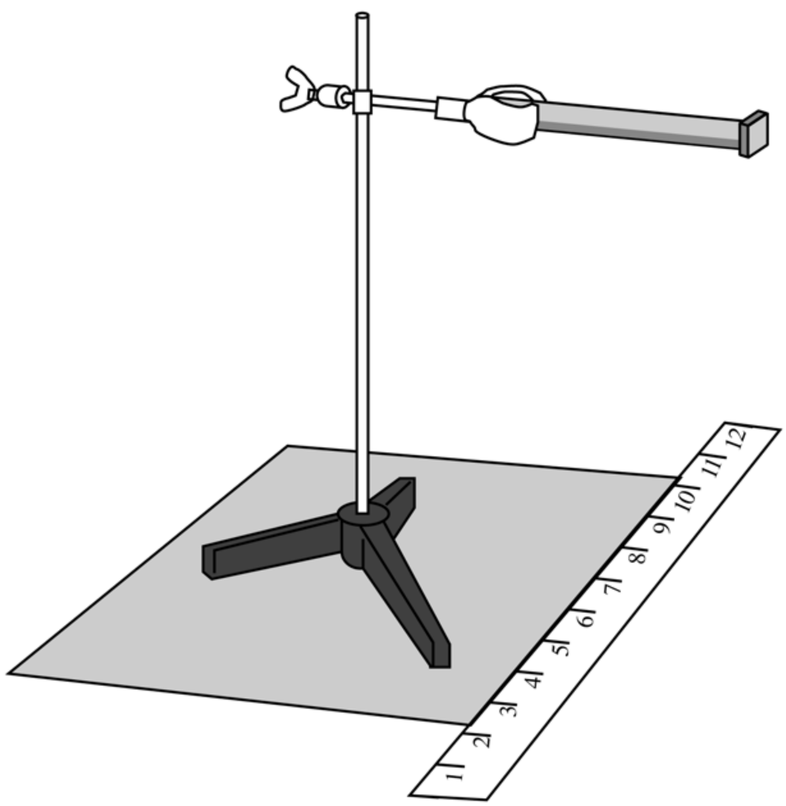


Figure 10.4

The dowel is made level using the spirit level for alignment. A voltage reading V_{1x} is taken. The dowel is then rotated so that the sensor is facing 180° opposite to the original orientation. A second voltage reading V_{2x} is taken.

The cardboard on which the ring stand rests is now rotated by 90°. The procedure used for the x-direction is repeated, so that the students obtain two voltages V_{1y} and V_{2y}. For the z-direction the probe is oriented so that the dowel is vertical, so that the Hall-effect device glued to its end is horizontal. The spirit level is again used to align the probe, and two readings V_{1z} and V_{2z} are taken.

The analysis of the data is simple. If the sensitivity of the gaussmeter is 50mV/μT, then the fields in the three directions are given by

$$B_x = (V_{1x} - V_{2x})/(50\ \text{mV}/\mu\text{T}) \quad (10\text{-}5a)$$

$$B_y = (V_{1y} - V_{2y})/(50\ \text{mV}/\mu\text{T}) \quad (10\text{-}5b)$$

$$B_z = (V_{1z} - V_{2z})/(50\ \text{mV}/\mu\text{T}) \quad (10\text{-}5c)$$

The magnitude of the Earth's magnetic field B_{Earth} is then given by

$$B_{\text{Earth}} = \left(B_x^2 + B_y^2 + B_z^2\right)^{1/2}$$

The dip angle ϕ of the field is given by

$$\phi = \tan^{-1}\left[B_z \Big/ \left(B_x^2 + B_y^2\right)^{1/2}\right]$$

LAB ELEVEN

Geometrical Optics:
Determining the Index of Refraction

Introduction

There are many ways to illustrate Snell's law in the classroom, and a large number of laboratory exercises can be used to determine the index of refraction. Here we present one method in which the students explore the use of simple geometry and Snell's Law in order to determine the index of refraction of a rectangular slab.

Snell's Law seems simple as it is presented in most textbooks. Consider a beam of light traveling in a medium that has an index of refraction n_1 striking a boundary at an angle of incidence α, as in Figure 11.1. If the second material has an index of refraction n_2, the beam of light leaves the boundary at an angle of refraction β, where β is given by Equation 11-1.

$$n_1 \sin\alpha = n_2 \sin\beta \qquad (11\text{-}1)$$

It is useful to mention to students that this equation may be derived by applying the condition that light travels in the shortest time from point A in medium 1 to point B in medium 2. While the application of calculus necessary to carry out the proof of this is beyond the requirements of the Physics B course, students should be apprised of this useful application of a minimum principle in physics. This helps raise physics beyond the application of an array of equations to the application of general principles that have validity and usefulness elsewhere in physics.

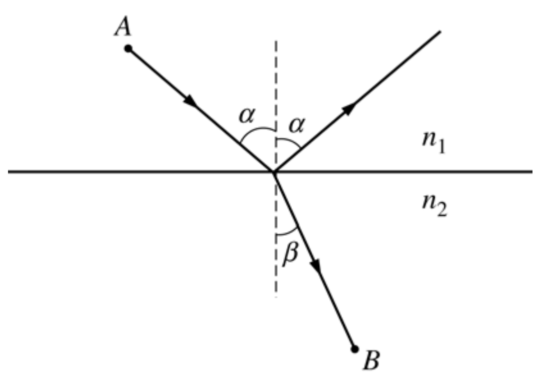


Figure 11.1

Theory

In this experiment the index of refraction of a rectangular slab is determined by measuring the distance the beam is displaced after it travels through the block, as shown in Figure 11.2. Here the slab has a thickness D, and the beam undergoes a transverse displacement S as it passes through the block. The problem for the students is to determine the index of refraction n_2 from measurements of D, S, and the angle of incidence α.

We first define a new distance R, which is the length of the path that the beam follows in the transparent slab. From Figure 11.2, we see that R is related to S by Equation 11-2.

$$S = R\sin(\alpha - \beta) \qquad (11\text{-}2)$$

This may be rewritten, using a trig identity, as

$$S = R(\sin\alpha\cos\beta - \cos\alpha\sin\beta) \quad (11\text{-}3)$$

From Figure 11.2 we see that *R* is related to *D* through Equation 11-4.

$$D = R\cos\beta \quad (11\text{-}4)$$

Substituting Equation 11-4 into Equation 11-3 to eliminate *R* yields Equation 11-5.

$$S = (D/\cos\beta)[\sin\alpha\cos\beta - \cos\alpha\sin\beta] \quad (11\text{-}5)$$

This simplifies to the following:

$$S = D(\sin\alpha - \cos\alpha\tan\beta) \quad (11\text{-}6)$$

This may be rearranged to Equation 11-7, which allows us to find β.

$$\tan\beta = \tan\alpha - (S/D\cos\alpha) \quad (11\text{-}7)$$

Once we have found the angle of refraction β, we may then find the index of refraction n_2 from Equation 11-8.

$$n_2 = n_1\sin\alpha/\sin\beta \quad (11\text{-}8)$$

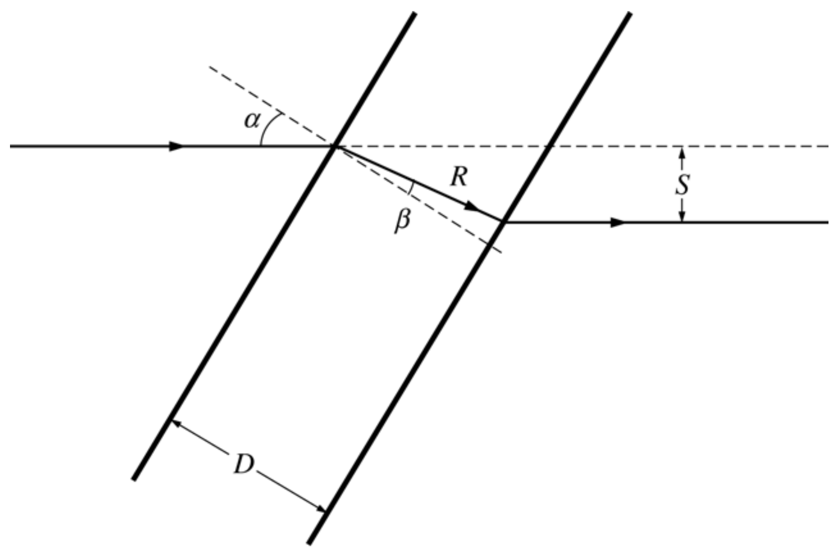


Figure 11.2

Apparatus

The experiment is carried out by sending a beam of light through a transparent block and measuring the transverse displacement *S*. The beam of light is best provided by a laser. The laser may be a laser pointer, although laser pointers often have a wide beam divergence on the order of several milliradians. A small **solid state laser** works better. Before turning on any of the lasers, the students must be instructed in the basics of laser safety. Many laser pointers have outputs of several mW, making them Class IIIa or IIIb devices. If the beam goes from the laser directly into the eye, severe and permanent eye damage may result. Students must understand that lasers are not toys, and that misuse may easily cause irreparable harm to others, or themselves. The laser should never be turned on unless it is firmly mounted on the lab bench, and directed at the transparent block. The laser pointer and block should be arranged along the long axis of the lab table, in such a way that makes it as difficult as possible for a student to place his or her face in the beam path. Obstacles, such as screens, should be placed in the path of the reflected and refracted rays, so the beams do not stray beyond the measurement area.

The students need a rectangular slab of transparent material. A **slab of glass 3 or 4 inches thick** works well, but may be difficult to find. **A shallow tank made of glass or acrylic that is filled with water** also works well**.** The tank, 3 or 4 inches on a side and an inch high, may be easily fabricated out of acrylic or glass pieces held together with waterproof cement.

The student needs some way of measuring the angle of incidence α. A simple **protractor** works well. The tank and protractor may be set on a piece of paper that rests on a **slab of Styrofoam**. The slab provides a backing into which **straight pins** are pushed to mark the location of the laser beam. The entire setup is illustrated in Figure 11.3.

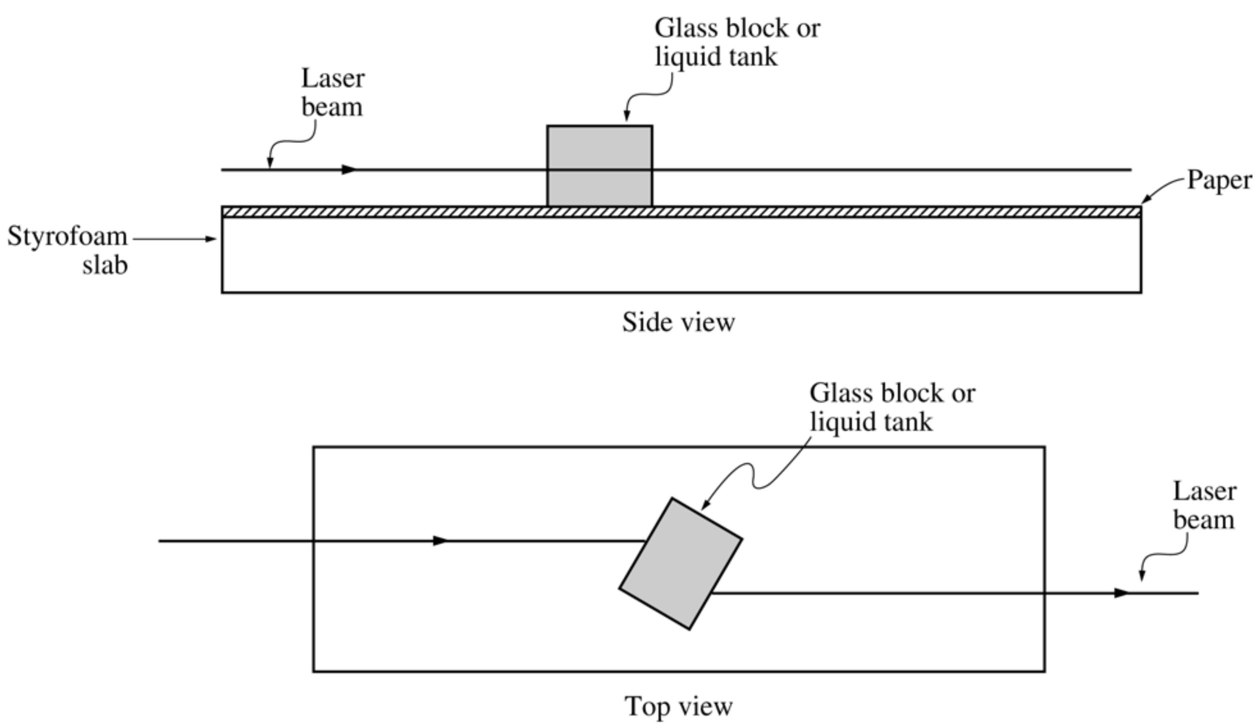


Figure 11.3

Experiment

The procedure consists of sending the beam through the slab at a number of angles, and marking the displacement of the beam. It is a good idea to tape the Styrofoam to the lab bench and pin the paper to the Styrofoam before taking any measurements. Once the laser is set up, the position of the beam should be marked on the paper by placing two pins in the paper. The pins should be placed so that the beam hits them before it gets to the slab. The pins should also be far enough apart so that it is easy to draw a straight line through them, and continue that line past the slab. A 4-inch separation is usually adequate.

The transparent slab and protractor are then placed on the paper in such a way that the beam goes through undeflected. This is used to mark the zero-degree angle of incidence. Two pins are used to locate the position of the beam after it leaves the slab.

The slab is then rotated to an angle of 30 degrees, and the new position of the beam after it passes through the slab is determined. Two pins are used to locate this new position of the beam. This process is continued by marking new beam positions after the slab is rotated to 40, 50, 60, and 70 degrees.

The angle of incidence is most easily determined by measuring the angle of deflection of the beam after it reflects from the slab's surface. The angle of incidence is one half of the angle of deflection, as shown in Figure 11.1. The reflected beam's location is also easily found using the same pin arrangement as for the transmitted beam. A typical set of data for a single reading is shown in Figure 11.4.

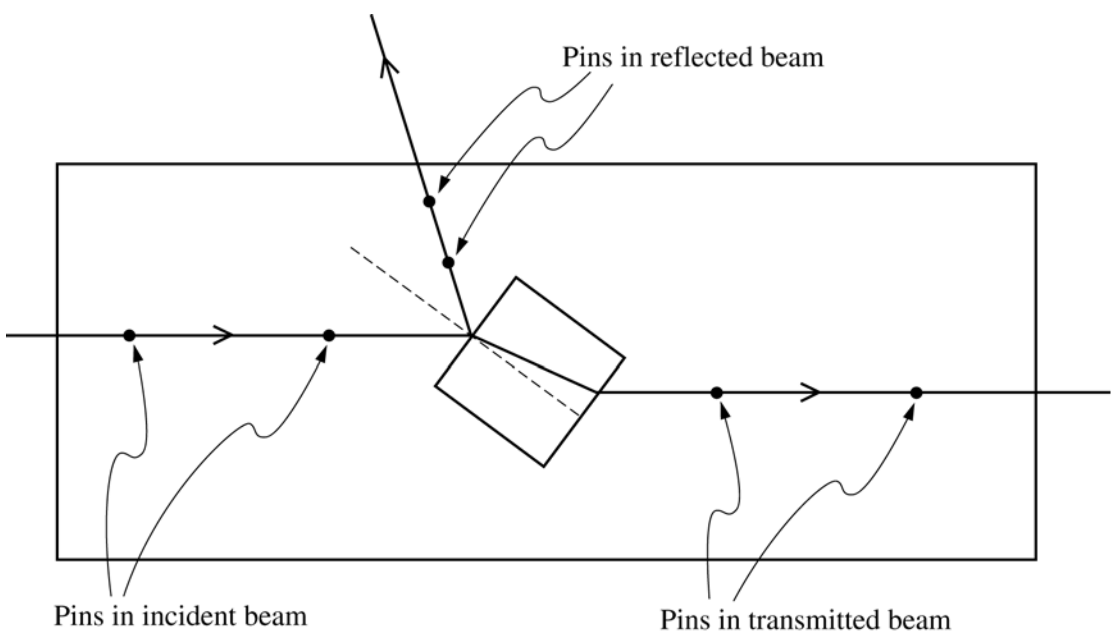


Figure 11.4

The sheet of paper with the pinholes in it is removed from the Styrofoam mounting board. The student then draws a line corresponding to the undeflected beam, and the five sets of straight lines corresponding to the transmitted and reflected beams. The student measures the angles of deflection in order to obtain the angle of incidence. The transverse displacement of the transmitted beam is measured to provide a set of five S values. The thickness of the transparent slab is measured and the student may proceed to the analysis.

Analysis

The data analysis can be carried out on a spreadsheet, but the presence of the trigonometric functions makes it just as easy to do by hand. Equations 11-7 and 11-8 are used to obtain values of n_2 from the data. The resulting n_2 values may then be compared with known values of the index of refraction for glass or water. Students should do some statistical analysis on their n_2 values, calculating an average value and a standard deviation. For students unfamiliar with the calculation of the standard deviation σ, it is given by Equation 11-9.

$$\sigma_x = \sqrt{\Sigma_i \left(x_{avg} - x_i\right)^2 \big/ (N-1)} \qquad (11\text{-}9)$$

Here N is the number of data points, x_{avg} is the average value of the variable x, and the x_i are the individual data values for the variable x.

After the class does the experiment, they should compare n_2 values and the experimental uncertainty in n_2. The values the students obtain should all agree to within $\pm 2\sigma_{n_2}$.

As an example, suppose a student obtains the following values for n_2 for water: 1.30, 1.35, 1.29, 1.30, and 1.36. The average value for these readings is 1.32. The standard deviation is 0.032, so that $n_2 \pm 2\sigma_{n_2}$ is 1.32 ± 0.06. This agrees with the standard value of 1.33, to within the experimental uncertainty.

Comments on Geometrical Optics

There are many kits that are available from scientific supply houses that allow the students to explore the properties of lenses and mirrors. If your class spends more than a week or two on geometrical optics, these kits are a worthwhile investment. If you spend only a couple of weeks on geometrical optics, this lab may provide your class with enough material in data analysis and experimental procedure.

LAB TWELVE

The Diffraction Grating

Introduction

Physical optics provides students with an opportunity to study the wave nature of light. The labs and demonstrations that the students encounter in physical optics allow them to better understand the concepts of modern physics that they encounter at the end of the Physics B course. One of the easiest experiments to perform in physical optics involves sending a monochromatic beam of light through a diffraction grating in order to determine the wavelength of the light source. Inexpensive diffraction gratings are available for less than a dollar apiece, and lasers, particularly in the form of laser pointers, are inexpensive as well. Red laser pointers cost less than $20. A green laser pointer, while more expensive at $200 or more, is useful to show the students how the dispersion of the grating depends on wavelength.

Theory

The theory of the diffraction grating is covered in most college physics textbooks. The example given usually begins with a two-slit aperture as shown in Figure 12.1. A beam of light is incident on the aperture, and each slit acts as a point source of light. The two sources give rise to an interference pattern on a screen that is a distance L from the apertures. We assume that L is much larger than the slit separation D. If the phase difference ϕ between the two waves is an even multiple of π, constructive interference results. Another way of stating this condition is that the optical path difference between the two sources is an integer number of wavelengths. From Figure 12.1 we may write this as

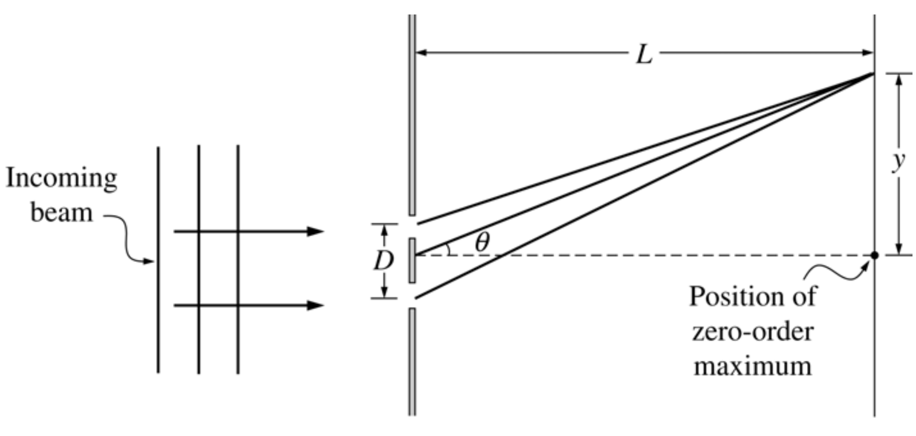


Figure 12.1

$$D \sin\theta = m\lambda \quad (12\text{-}1)$$

where m is an integer (0,1,2,3,4, ...) and λ is the wavelength of the light. We may rearrange this equation to

$$\sin\theta = m\lambda/D \quad (12\text{-}2)$$

If we wish to locate the maximum of order m on the screen in terms of distance y from the zero-order maximum, as shown in Figure 12.1, we may use

$$y = L \tan\theta \qquad (12\text{-}3)$$

and then use Equations 12-2 and 12-3 to locate y. Students should note that as the wavelength λ increases, the angle θ increases as well. The dispersion of a grating is thus opposite to that of a prism, in which long wavelengths undergo less deflection than short ones. It is useful to demonstrate this behavior to the class using a clear incandescent light bulb, a prism and a grating so that they may understand this fundamental difference between the two dispersive optical components.

Experiment

Lab apparatus consists of a **diffraction grating** mounted in a holder such as a ring stand clamp, a sheet of paper used as a screen, and two or more sources of monochromatic light. The sources may be a **HeNe laser**, a **red laser pointer**, and a **green laser pointer or laser**. The HeNe laser has an output at 633 nm, while the red laser pointer has an output around 650-670 nm, depending on the laser diode used in the pointer. The green laser pointer has an output around 530 nm. The students use the HeNe laser to calibrate their gratings, and then use the gratings to determine the wavelength of the output of the laser pointers.

Before turning on any of the lasers, the students must be instructed in the basics of laser safety. Many laser pointers have outputs of several mW, making them Class IIIa or IIIb devices. If the beam goes from the laser directly into the eye, severe and permanent eye damage may result. Students must understand that these things are not toys, and that misuse may easily cause irreparable harm to others, or themselves. The laser should never be turned on unless it is firmly mounted on the lab bench, and directed at the grating. The laser pointer and grating should be arranged along the long axis of the lab table, in such a way that makes it as difficult as possible for a student to place his or her face in the beam path.

Once the grating, laser, and screen are set up, the students may turn the laser on and observe the interference pattern that results. Again, care should be taken to eliminate stray laser beams bouncing around the room. The beam is weaker after it passes through the grating, but can still cause eye injury. Students should be cautioned to not look at the apparatus until they are sure that a scattered or reflected portion of the beam will not hit them in the eye. One way of doing this is to look away from the apparatus as it is turned on, then slowly moving their head back and forth and looking to see if a bright spot appears on the wall in front of them when they move their head. If a bright spot appears, that means that the laser beam is passing through the space where their head was. The apparatus should be turned off, and realigned to eliminate the stray beam. It is useful to mount the gratings so that the interference pattern appears along a vertical, rather than a horizontal axis, as this lessens the danger of stray laser beams in the lab.

The grating calibration should be done at one lab station in the classroom. Students bring their gratings to the lab station and mount the grating in the holder. They should tape or otherwise mount a piece of paper on a screen about 1 meter away from the grating. They mark the position of the zero-order maximum and the first-order maximum on the paper after the laser is turned on. The students need to determine how they mark the position of two maximums. They may choose to attempt to determine the center of the maximums, or to draw a circle around the maximum. They should describe whatever procedure they use in their lab report, and explain why they choose that procedure.

Once the maximums are located, the students then go to their own lab station and set up their laser pointer in the same way the HeNe laser and grating were set up. The students shine the laser beam through the grating, and mark the location of the zero-order and first-order maximums on a sheet of paper. They use Equations 12-2 and 12-3 to calculate D from the known wavelength of the HeNe laser, and then may calculate the wavelength of the output of the laser pointer.

Analysis

The students should carry out an error analysis in this lab to determine the accuracy with which they know the wavelength of the laser pointer. The sources of error are the measurements that the students use to determine the slit separation in the grating, and the measurements of the maximum of the interference pattern from the laser pointer. Here we present an example of the error analysis that may be done to complete this lab.

We begin with the determination of the slit separation in the grating. Suppose the screen is measured to be 0.950 meters from the grating, with an error of ± 0.002 meters. The distance between the zero-order maximum and the first-order maximum is measured to be 0.355 meters. The spots were 0.002 meters wide, so we assume that the error associated with this measurement is 0.004 meters. We then apply Equation 12-3 to calculate the angle θ. The minimum value of the angle occurs if y is 0.351 meters and L is 0.952 meters. Equation 12-3 then yields an angle of 20.2°. The maximum value of the angle occurs when y is 0.359 meters and L is 0.948 meters. The angle is then 20.7°.

Using Equation 12-2, we take λ as 633 nm. Inserting the minimum θ value in Equation 12-2, we obtain a D value of 1.83×10^{-6} m. Inserting the maximum θ value in Equation 12-2, we obtain a D value of 1.79×10^{-6} m. We take the average D value to be 1.81×10^{-6} m, with an uncertainty of $\pm 0.02 \times 10^{-6}$ m.

We then take the measurements for the laser pointer interference pattern. Suppose we found that the distance between the zero-order maximum and the first-order maximum was 0.370 meters, with the same uncertainty of ± 0.004 meters. The screen was 0.970 meters, with an uncertainty of ± 0.002 meters. Using Equation 12-3 as before, these data yield minimum and maximum angles of 20.6° and 21.1°, respectively.

Now we return to Equation 12-2. Using the minimum value of D $\left(1.79 \times 10^{-6} \text{ meters}\right)$ and the minimum value of θ $\left(20.6^\circ\right)$, we obtain a wavelength of 629 nm. Using the maximum value of D $\left(1.83 \times 10^{-6} \text{ meters}\right)$ and the maximum value of θ $\left(21.1^\circ\right)$, we obtain a wavelength of 659 nm, so the wavelength may be reported as 644 ± 15 nm.

In fact, this procedure overestimates the error in the experiment. But this procedure, as crude as it is, gives the students a more immediate feel for the error involved than if we hand them a set of formulas, particularly formulas involving the derivatives of trigonometric functions, that are necessary for a more accurate estimate of the error.

LAB THIRTEEN

Planck's Constant on the Cheap

Introduction

In 1899, when Planck put forward the hypothesis of a quantum of action, he did not envision it as a fundamental quantity. Rather, it was a parameter that conveniently led to the correct description of the blackbody spectrum. Today we see it much differently, and consider it to be one of the fundamental constants of nature.

Niels Bohr later used Planck's constant in his explanation of the energy levels of the hydrogen atom. One of the postulates Bohr used in his seminal 1913 paper was that the energy of the light quantum given off by an atom when the electron makes a transition from an initial state E_1 to a final state E_2 is given by

$$hf = E_1 - E_2 \qquad (13\text{-}1)$$

where f is the frequency of the light emitted. Equation 13-1 is the cornerstone of atomic and molecular spectroscopy. By measuring the frequency of light emitted by an atom or molecule, we can build up a model of the energy states that are accessible to its electrons.

In this lab we work in reverse from the usual spectroscopic procedure. Here we have an electron of charge e undergo a transition as it moves through a potential drop V. In a light-emitting diode (LED) the electron emits a photon as it moves through this potential drop. Following Bohr's reasoning, we write that the frequency f of the light emitted during this transition is related to the potential drop V by

$$hf = eV \qquad (13\text{-}2)$$

where e is the charge on the electron. We can easily rearrange this to

$$h/e = V/f$$

$$hc/e = V\lambda \qquad (13\text{-}3)$$

where c is the speed of light, and λ is the wavelength of the light emitted by the LED. By measuring the voltage drop through which the electron falls, along with the wavelength of the emitted light, we obtain a quantity equal to the three fundamental constants on the left side of Equation 13-3.

Apparatus

The apparatus required for this lab is straightforward to assemble, and the data easy to acquire. The circuit shown in Figure 13.1 is used to carry out the experiment. The power supply can be any combination of batteries from 3 to 9 volts, such as a couple of D cells, a nine-volt battery, and so on. R_1 is a 1-kΩ, $\frac{1}{4}$-watt resistor that limits the current through the diode. The diode with two arrows emanating from it is the LED. The voltage drop across the diode is measured with M_1, an inexpensive digital multimeter (DMM).

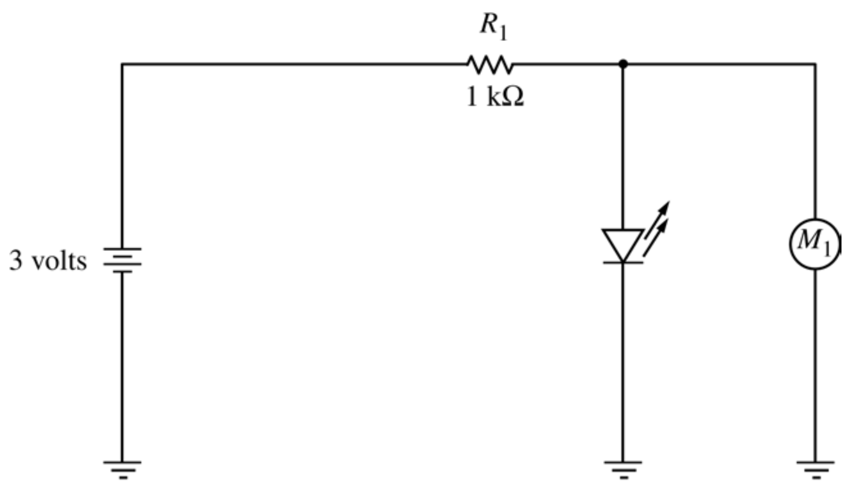


Figure 13.1

One way to do this lab is by using the circuit shown in Figure 13.1 and an assortment of LEDs. The LEDs should have outputs ranging from blue down to infrared. Cheap LEDs are available with output wavelengths in the infrared, red, yellow, green, and blue regions of the spectrum. A selection of five LEDs will yield five different voltage drops V that may be plotted against wavelength. If we rearrange Equation 13-2 we may obtain

$$\frac{h}{e} f = V \qquad (13\text{-}4)$$

so that a plot of V versus f should yield a straight line with a slope of h/e.

The obvious next question is where do the students get the wavelengths for the diodes. The short answer is that the wavelengths are often printed on the LED packaging for diodes available through retailers such as Radio Shack. A longer perhaps more satisfying answer is that students can guess the wavelengths for the LEDs with outputs in the visible by using the depictions of the visible spectrum often found in textbooks or science classroom wall charts. Before you eschew this approach, recall that the LED output is usually stated to within ± 10 nm. Also, the LED output is not monochromatic. The wavelength listed on the LED package is the wavelength at which the intensity of the light output is highest. A plot of light intensity versus wavelength is peaked at that wavelength, but it has a full-width at half-maximum of around 20 nm as well. (Full-width at half-maximum refers to the width, in nanometers, of the intensity distribution at one-half of the maximum value of the intensity.) Experience shows that the students' guesses are actually accurate to within ± 20 nm, and having the students guess the wavelengths does not affect the accuracy of the results. It is more useful to have the students guess, rather than to have them read a number from the package. Of course, if you use an infrared LED, that wavelength can be obtained from the package.

If the lab is particularly well equipped and has a spectrometer available, the students may obtain the intensity versus wavelength curve described above for themselves. That is unnecessary for the purpose at hand, and may raise more questions than it answers for the students. A full description of the processes that give rise to the observed spectrum is usually not encountered in a college-level course until the junior year in a solid-state physics course.

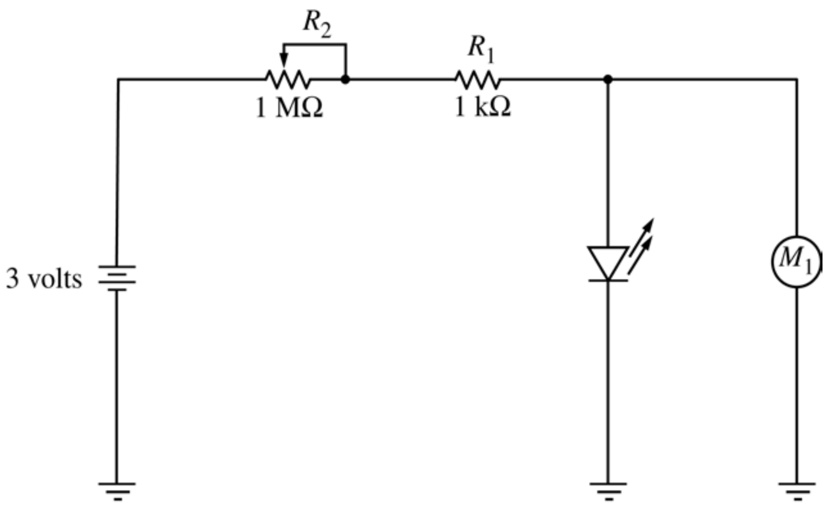


Figure 13.2

Another approach using a single LED is shown in Figure 13.2. The only change from the circuit in Figure 13.1 is the addition of R_2, a 1-MΩ variable resistor. In addition, the power supply should be a pair of *D* cells. The resistor R_1 is retained so that students cannot accidentally burn out the LED by turning the variable resistor to zero.

This experiment is best undertaken in a dark room. (Wire up the circuit before turning off the lights.) The variable resistor R_2 is adjusted until the LED barely glows. The potential *V* is read from the DMM, and the wavelength determined as above. It is best to turn the room lights up and adjust R_2 until the LED glows brightly before determining the wavelength. The reason for adjusting the brightness of the LED is to obtain a voltage reading that corresponds to the minimum energy required to generate a photon. While this technique reduces the error associated with the voltage measurement, the error associated with the measurement of the wavelength is still present.

After the voltage and wavelength data is acquired, the frequencies associated with the measured or guessed wavelengths are calculated. The voltages are plotted against the frequencies, and a best-fit straight line is drawn through the data. The slope of the straight line should equal h/e to within 10%. This is as well as one does with a commercial Planck's constant apparatus costing far more than the equipment described here. In addition, if the students do a least-squares fit of the data, they usually find that the deviation of the slope from the nominal value of h/e is less than the uncertainty associated with the slope.

LAB FOURTEEN

The Photoelectric Effect*

Introduction

This experiment involving the photoelectric effect provides the student with an excellent opportunity to investigate one of the primary pieces of evidence that was used to support quantum theory in the early 20th century. The textbook descriptions of the effect often gloss over the difficulties of carrying out the experiment. The photocurrents produced in the lab are usually less than a milliamp, and require a stable and sensitive amplifier to be detected. Here we present an inexpensive circuit that can be built for less than $150 per lab setup that enables the students to investigate several different aspects of the photoelectric effect.

Theory

The photoelectric effect is covered in detail in most introductory textbooks that contain chapters on modern physics, but a brief review is given here. When a photon of energy hf is absorbed by an electron in a metal, the electron may escape the metal if the energy of the photon is greater than the work function W of the metal. If $hf > W$, then the electron acquires a kinetic energy K given by

$$K = hf - W \qquad (14\text{-}1)$$

If the photocathode that emits these electrons is placed in a vacuum, the electrons can be attracted to a positively charged anode as shown in Figure 14.1. The ammeter A measures the current due to the electrons that are ejected by the photocathode. To give some idea of how small the current is, consider a 1 mW, 633 nm laser shining directly on the photocathode. If every photon ejected an electron upon striking the photocathode, the photocurrent would be approximately 500 microamps.

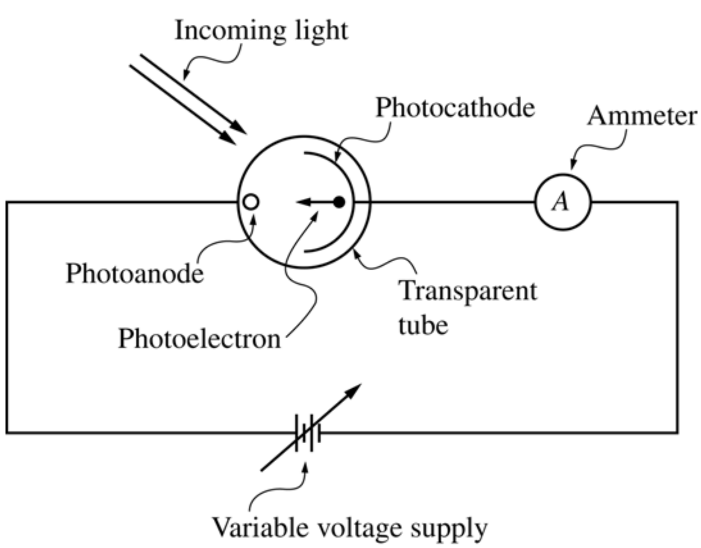


Figure 14.1

* This lab requires the teacher or students to construct some electronic apparatus. Teachers with little or no experience with electronics may wish to forego this lab, or to undertake it as an extended project with students.

If we now apply a negative potential to the photoanode, the photoelectrons are slowed as they approach it. We may increase the magnitude of the negative potential until no electrons have enough kinetic energy to reach the photoanode. The magnitude of the electric potential necessary to stop all photoelectrons from reaching the photoanode is the stopping potential V_{stop}. The most energetic photoelectrons have a kinetic energy K that is equal to the product of the charge of the electron e and the stopping potential V_{stop}. We may write Equation 14-1 as

$$eV = hf - W \qquad (14\text{-}2)$$

We may illuminate the photocathode with light of a particular frequency and then measure the stopping potential. After repeating this procedure for several frequencies, we may then graph the stopping potential versus the frequency, and obtain a straight line with a slope h/e. The y-intercept of that straight line is equal to the work function of the metal.

Apparatus

A. The Electronics

A block diagram of the electronics used in this experiment is shown in Figure 14.2. The phototube may be forward-biased or reverse-biased. When the phototube is forward-biased, the

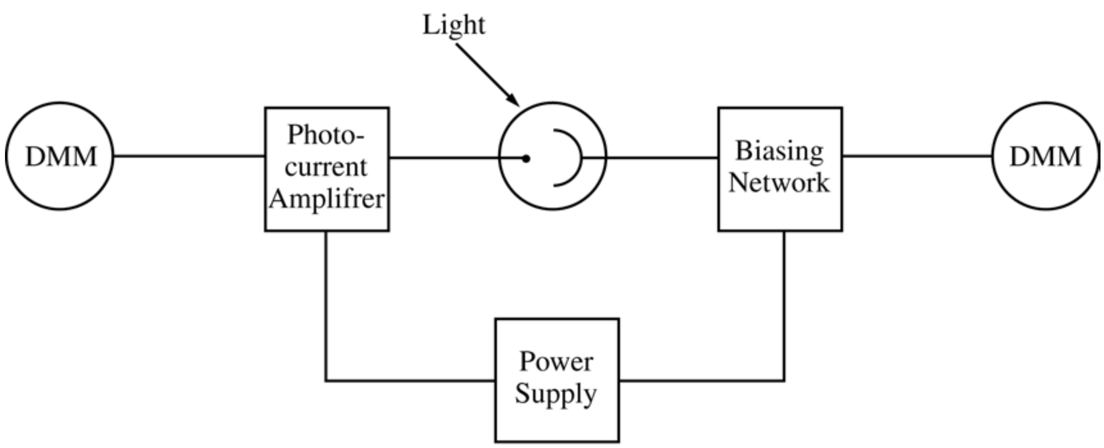


Figure 14.2

photocathode is electrically positive with respect to the photoanode. When the phototube is reverse-biased, the phototube is electrically negative with respect to the photoanode. Both configurations are used in this lab. The reverse-bias configuration is the configuration used to determine the stopping potential.

The photocurrent is measured by an operational amplifier (op amp) that is wired up as a transconductance amplifier, as shown in Figure 14.3. The choice of op amp is not critical, but it should be one that is stable over time with low noise. An OP-7 is a good choice, and any similar op amp should also do well in this circuit. The output of the transconductance amp goes to a digital multimeter, used as a **voltmeter**. An output of one volt corresponds to a photocurrent of 1 microamp. The circuit construction may be done on a solderless breadboard. Wiring the circuit on a small piece of perf board and mounting it in a metal box may be preferable, as it reduces set-up time significantly. The op amps should be mounted in 8-pin DIP sockets, as soldering directly to the pins on the op amp may damage the devices.

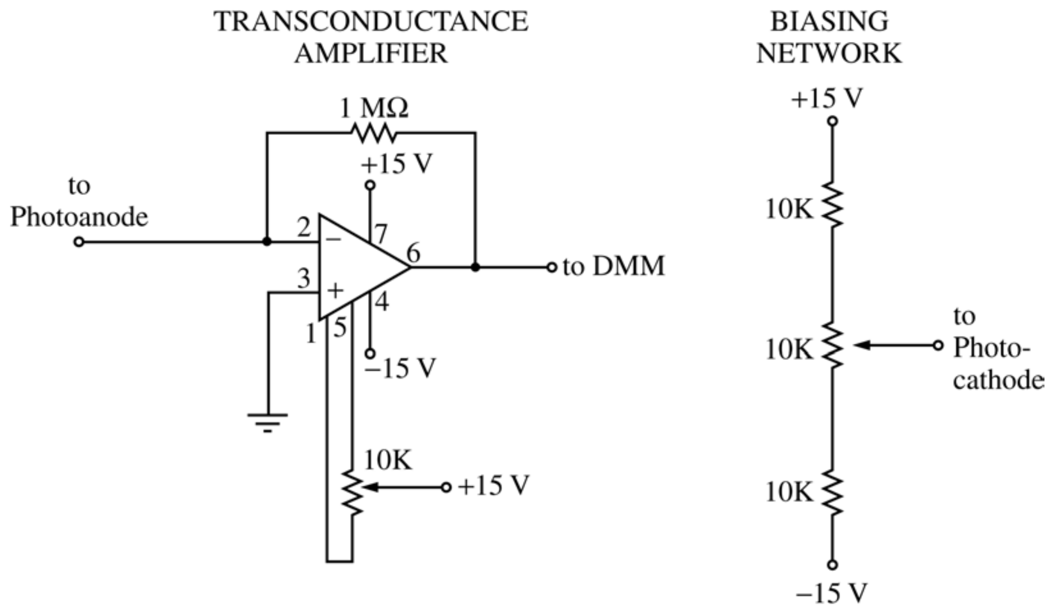


Figure 14.3

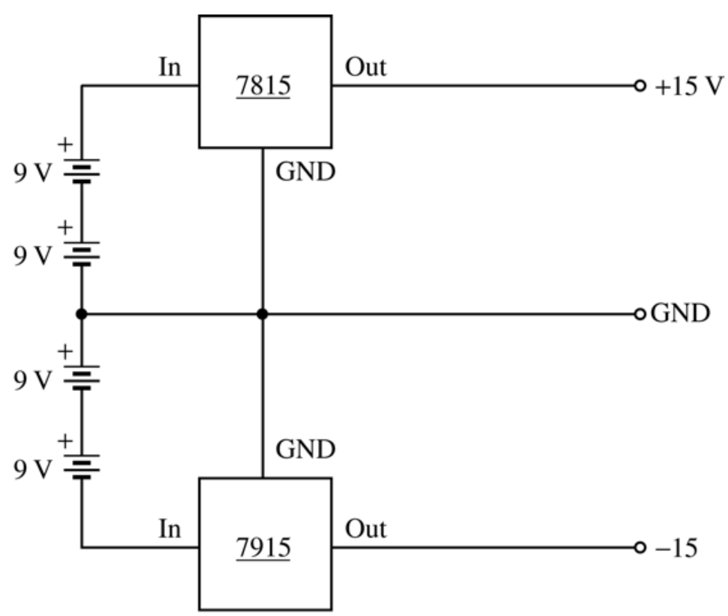


Figure 14.4

The power supply is a simple ± 15-volt split supply, as shown in Figure 14.4 and 14.5. If you don't plan to mount the circuit in a metal box, the circuit shown in Figure 14.4 may be used. The disadvantage to the circuit shown in Figure 14.4 is that it uses four 9-volt batteries. The advantage is that it is safer than the circuit shown in Figure 14.5.

IF YOU USE THE CIRCUIT SHOWN IN FIGURE 14.5, YOU MUST MOUNT THE POWER

SUPPLY IN A GROUNDED METAL ENCLOSURE. USING 120VAC ON A SOLDERLESS BREADBOARD IS EXTREMELY HAZARDOUS, AND MAY RESULT IN SERIOUS INJURY OR DEATH. Given that, you may wonder why anyone would opt to use the circuit shown in Figure 14.5. The reason is that it does away with batteries, which have a nasty habit of going dead while you or the students are taking data, and it is cheaper to operate.

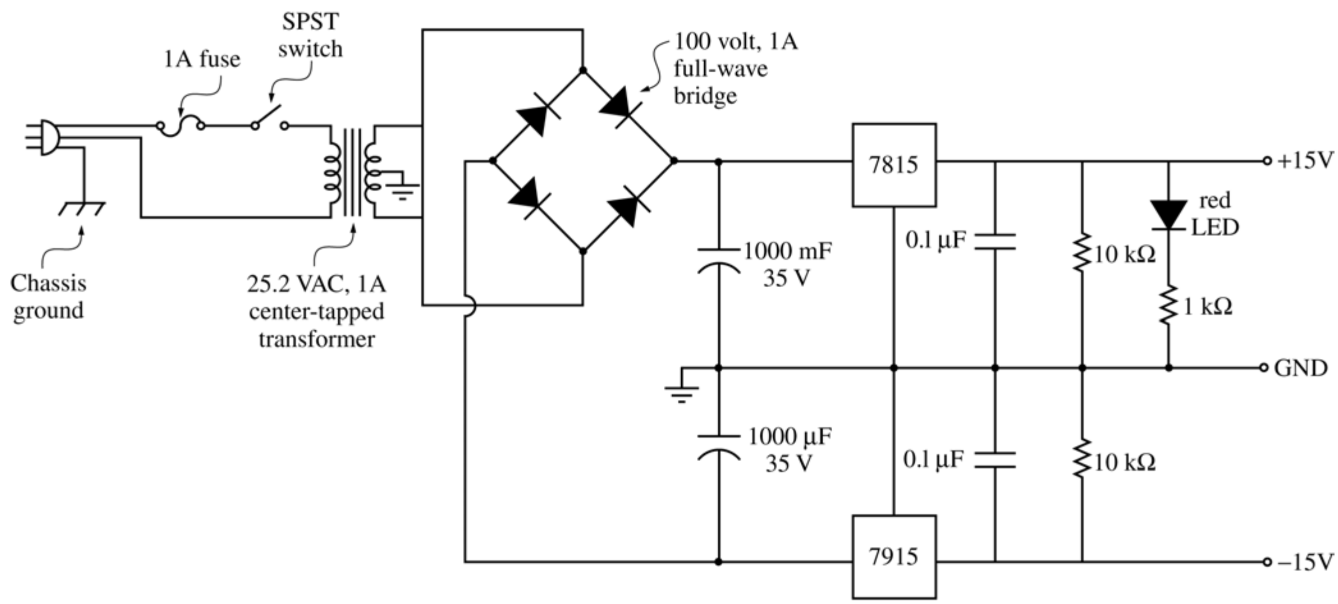


Figure 14.5

For all of the integrated circuit components used in this lab, it is imperative to follow the pin-out given in Figure 14.6. Please note that the pin-outs on the 7815 and the 7915 voltage regulators are different.

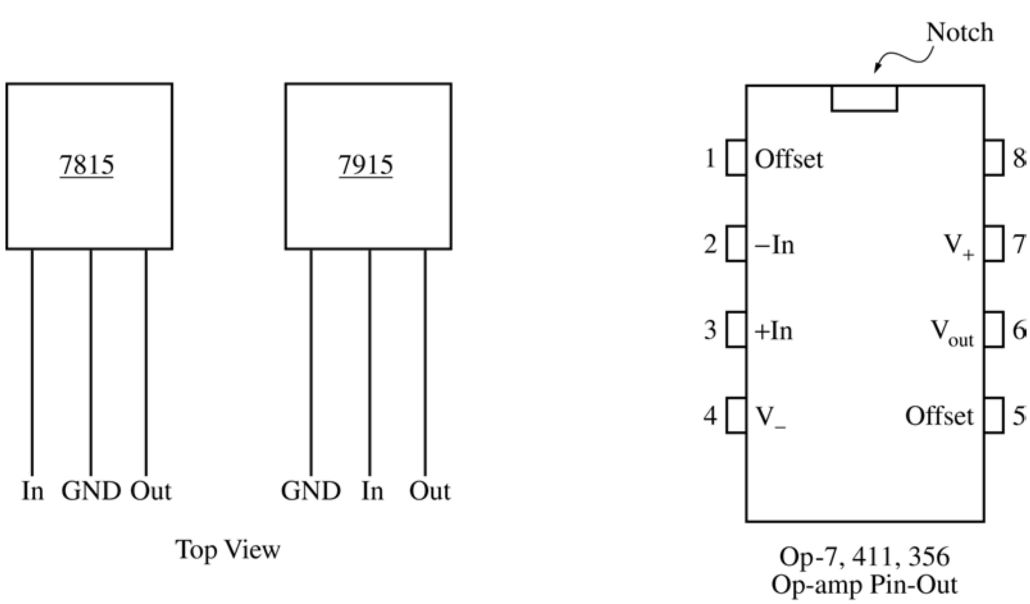


Figure 14.6

B. The Phototube

The phototube is a 1P39 mounted in a ceramic octal socket. One nice feature of the phototube is that it has a clear glass envelope, and students can easily see the photocathode (the large curved electrode) and the photoanode (the upright wire in the center of the tube). A pair of color-coded twisted leads runs from the ceramic socket to the amplifier.

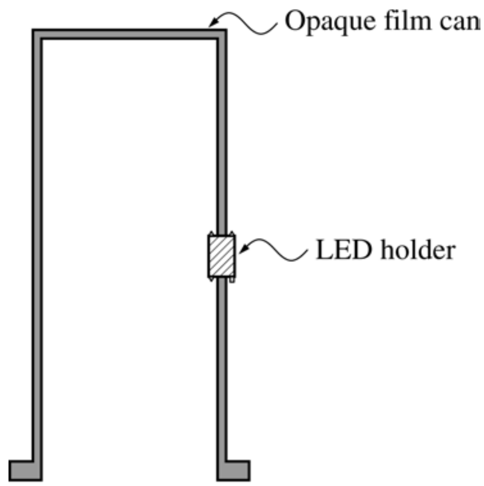


Figure 14.7

The easiest way to keep out stray light is by slipping an opaque plastic 35mm film canister over the phototube. Before slipping it over the tube, a $1/4$ inch hole should be drilled in the film can about halfway down the side of the canister. A plastic LED holder, available at most electronic parts dealers, is then placed in the hole so that an LED may be mounted to shine on the phototube, as in Figure 14.7.

The determination of the wavelength of the LEDs is described in the *Planck's Constant on the Cheap* experiment, and is not repeated here. Blue, green, yellow, red and infrared LEDs may be used. The light from the 912-nm LEDs does not generate a photocurrent, as the energy of the infrared photons is less than the work function of the photocathode. Explaining this absence of photocurrent brings home to students one of the puzzling aspects of the photoelectric effect, that is, that no photocurrent is generated for light whose energy is an amount hf below the work function.

Experiment

The acquisition of the data is straightforward. Once the LED is inserted in the $1/4$ inch LED holder and the electronics are turned on, the photodiode should be forward biased to its maximum voltage. This should be around 5 volts. One DMM monitors the bias voltage, while the second DMM monitors the output of the transconductance amplifier. Remember that a one-volt output of the amplifier corresponds to a photocurrent of one microamp.

The bias voltage is decreased to zero in half-volt increments. At each increment the photocurrent is recorded. When the bias voltage reaches zero, the bias switch is thrown to reverse the bias on the phototube, and the procedure is repeated. The data exhibit a decrease, but it is not possible to find the stopping potential exactly using this method. It does allow the students to find the stopping potential to within a volt or so. Remember that the stopping potential is the bias voltage at which the photocurrent goes to zero.

Once the stopping potential is located approximately, the students should approach it in 0.1 volt increments from at least ± 1 volt away. For example, if it seems the stopping potential is 1.2 volts, the students should go to zero volts and increase the voltage 0.1 volt increments. As the stopping potential is approached, the output of the amplifier will fluctuate, making it difficult to find the point where the photocurrent is zero. If the photocurrent is plotted versus the bias voltage, a best-fit straight line to the data taken close to the stopping potential is one method of getting around this difficulty.

This procedure is then repeated for the other LEDs. Once the stopping potentials are found, they may be plotted versus the frequency of light emitted by the LEDs. This graph then yields h/e, and the work function of the photocathode.