ACP 2025 RC CIRCUITS AND MORE
TEACHER NOTES

DESCRIPTION
A few simple electronic components can illustrate some good principles of physics. We will use this segment of a circuit:

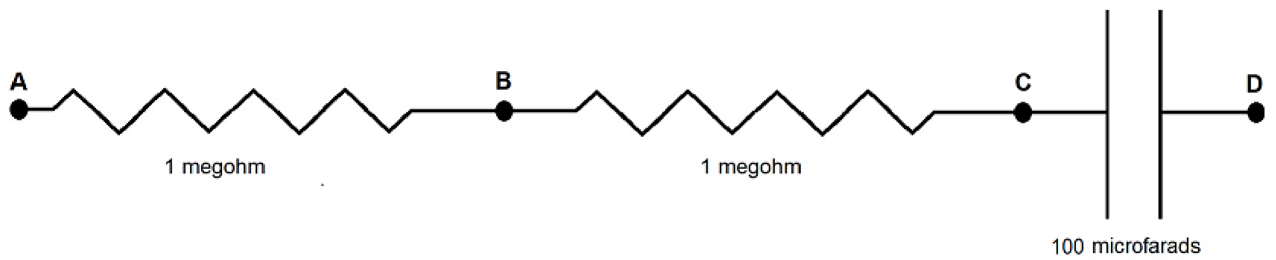

Figure 1. Circuit elements in this activity.

What you see is two resistors and a capacitor arranged in series. We can use this to study resistors in series or parallel and to investigate the discharge of a capacitor. This last part will be our primary focus.

By connecting a 9 v battery to points C and D, we can rapidly charge the capacitor to 9 v. If we disconnect the battery and then connect A and D with a wire, the capacitor will discharge more slowly. Measuring the voltage on the capacitor at points C and D with a meter, we can record the voltage as a function of time to show its exponential decay and find the time constant. This process is mathematically identical to the decay of unstable particles, as we will see.

STANDARDS ADDRESSED
Please apply relevant physics teaching standards.

ENDURING UNDERSTANDING
Exponential functions are a key feature of natural processes.

LEARNING OBJECTIVES
As a result of this activity, students will be able to:

- Connect a simple circuit.
- Measure voltage.
- Plot and analyze an exponential decay.
- (In addition) Explain how series and parallel circuits give different net resistances.

PRIOR KNOWLEDGE
Students should be able to:

- Use a stopwatch.
- Take good notes for data.
- Read a meter.
- Make a plot from data.

MATERIALS

- Two resistors, 1 megohm
- One capacitor, 100 microfarad
- Three wires with alligator clips on each end
- 9 volt battery
- Digital or analogue multimeter
- Notepad.

RESOURCES

IMPLEMENTATION

Students should connect the two resistors and the capacitor as shown in Figure 1. Students can then connect the battery across the capacitor to charge it. The capacitor is electrolytic, giving it preferred positive and negative sides. The short lead is negative and the long lead is positive. The larger terminal of the battery is negative and the smaller terminal is positive. Connect positive-to-positive, negative-to-negative for charging. See Figure 2 below.

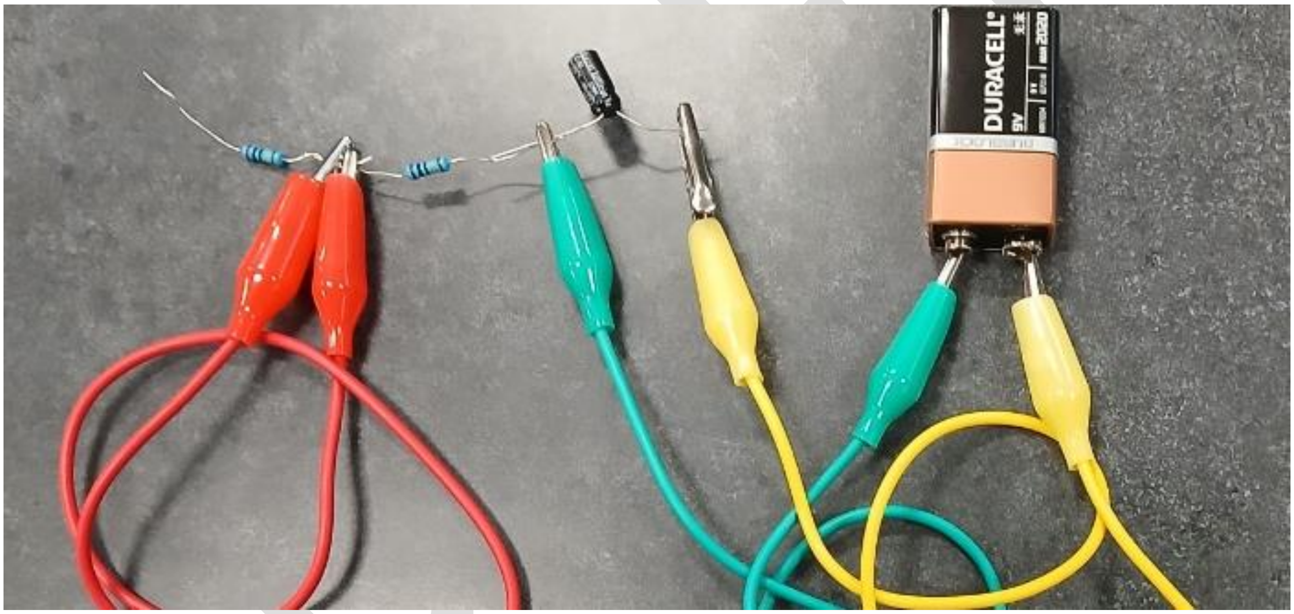

Figure 2. Charging the capacitor.

Note that we can use alligator clips to connect leads to make the circuit shown in Figure 1.

The next step is to connect points A and D from Figure 1 (the left lead of the capacitor to the left to the negative lead of the capacitor) with the battery removed. As quickly as possible, begin to measure the voltage across the capacitor, taking one reading at a fixed time interval (10 seconds is suggested). This should take at least three students: a timer, a multimeter operator, and a voltage recorder for each interval. Please see Figure 3 below.

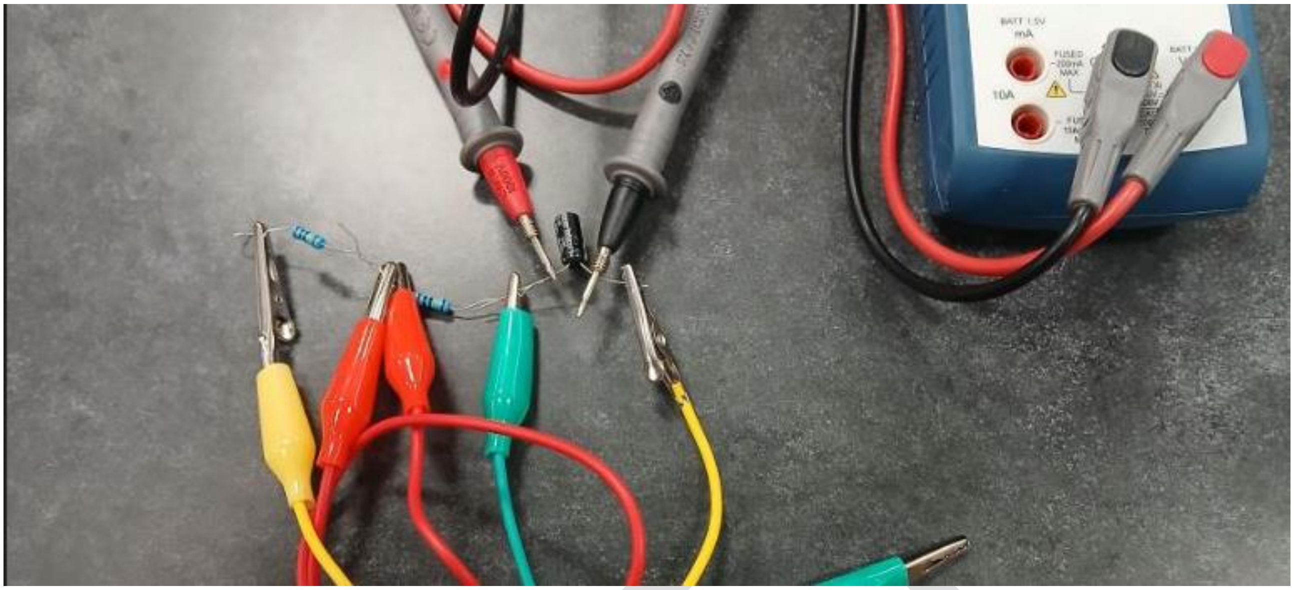

Figure 3. Connections to measure capacitor voltages.

The results should be recorded in a table as in Figure 4. It does not need to be a spreadsheet: paper is fine.

Temps (s)	Tension (v)
0	
10	
20	
30	
40	

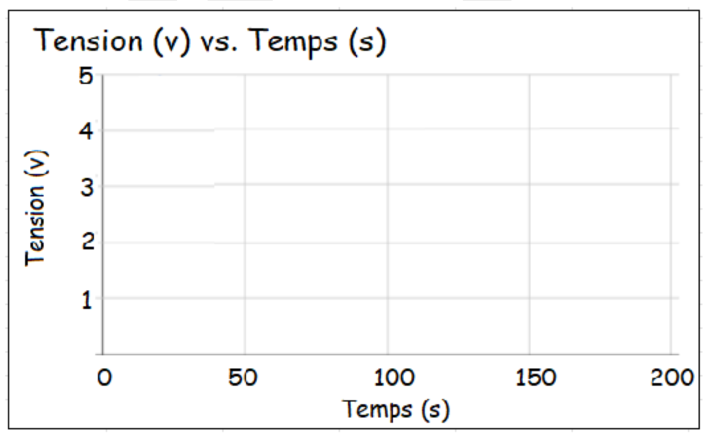

Figure 4. Sample data table and graph template.

The figure shows the beginning of the table and a blank template of the data plot. Students should take data for at least 200 seconds. More is better.

Expected Result and Discussion

The expectation is that the student group will each find that the voltage decreases exponentially over time according to the equation

$$\mathbf{V = V_o\, e^{-t/\tau}}$$

where V is the voltage across the capacitor, V_o is the initial voltage from when measurement begins, t is time in seconds, and τ is the time constant of the decay. The letter e = 2.71828… is the base of natural logarithms.

In this formulation, after time τ, the voltage V should have reduced to from V_o to V_o/e. The time constant is given by τ = RC.

In this activity, students can find τ by dividing the initial voltage by e and then reading the corresponding time in the plot. In practice, this may not be exactly equal to RC. In Figure 5, we see a sample result (from actual measurement) and accompanying calculations.

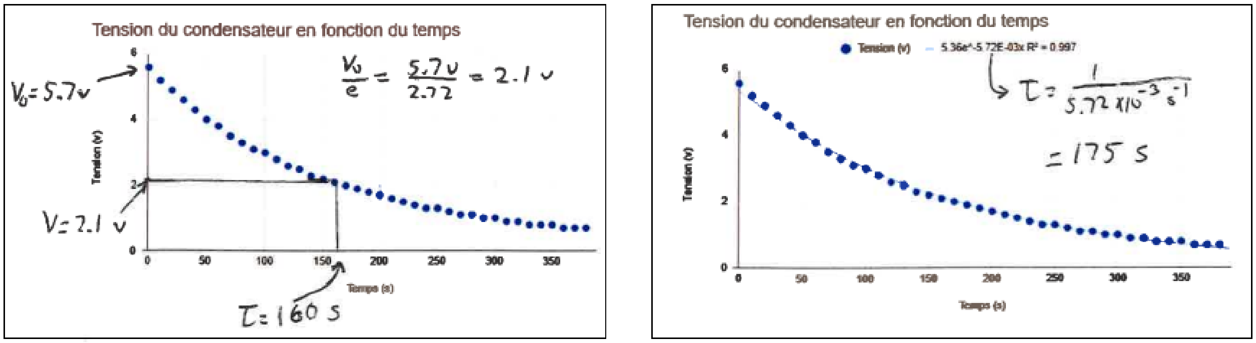

Figure 5. Sample results.

On the left, we see the raw plot with all work done by hand. The Initial voltage V_o is measured to be about 5.7 v. Dividing by e = 2.72 (rounded), we get V = 2.1 v. We then draw a horizontal line to the plotted points and, from there, draw a vertical line down to the horizontal axis. We read from there that $\tau = 160$ s.

On the right, we have added a trendline and equation using Google sheets. The equation reads

Tension (v) = 5.36E-5.72x with a correlation coefficient of 0.997. (Good fit!)

Using our notation in these Notes, that reads

$V = V_o e^{-0.00572t}$ in which $\tau = 1/0.00572\ s^{-1}$. Thus $\tau = 175$ s.

A direct calculation of the time constant from the values of the components gives

$\tau = RC = 2(1x10^6\ \Omega)(1x10^{-4}\ F) = 200$ s.

This does not include the tolerances of the components or uncertainties in a very quickly assembled measurement, however. Let's see if students can come closer.

The main point and the direction to guide discussion, however, is the negative-exponential behavior of the discharge of the capacitor. Analogous behavior is found widely in nature. Atomic, nuclear, and particle decay follow the same mathematical form. Examples:

- Half-life in carbon dating is based on the exponential decay of ^{14}C atoms; with half-life, the number 2 is substituted for e in the calculation, where the half-life is the time it takes for half the ^{14}C atoms to decay to ^{12}C.
- Lifetime in nuclear or particle decay is calculated exactly as the RC time constant is. Looking at data from decay of muons in a cosmic ray detector (https://bit.ly/467OevD) shows the same form.

The reason is that the discharge of a capacitor is, like the examples above, governed by probability. There is some number of electrons stored in the negative plate. When they have a pathway (a completed circuit), electrons will individually and randomly leave the negative plate, follow the circuit, and join the positive plate. This will occur until both plates are neutral. The greater the resistance in the circuit and the more electrons stored due to the capacitance of the capacitor, the greater the time constant and thus the longer it takes for 1/e of the electrons to flow to the other side. Thus $\tau = RC$.

A sample of ^{14}C atoms will also randomly and individually decay to ^{12}C. The decay curve, using mass of ^{14}C and time in years, will look like our RC circuit discharge plot. So will the decay of particles from muons to all sorts of mesons, this time with numbers of particles and very short times in microseconds, nanoseconds, or even shorter times.